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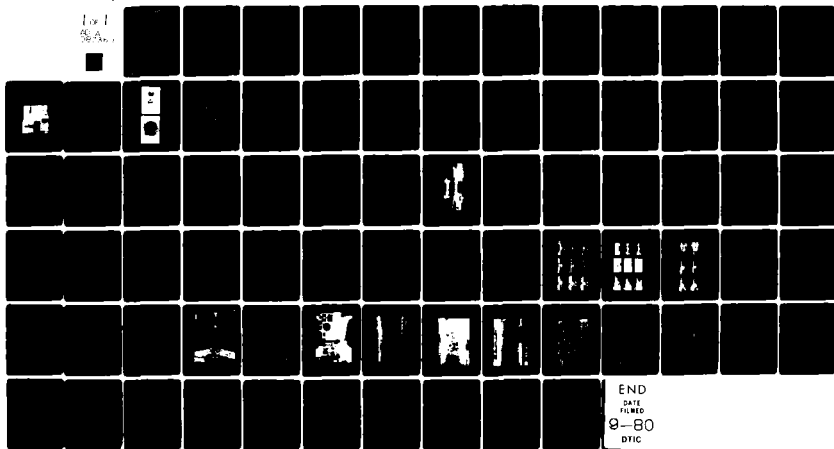
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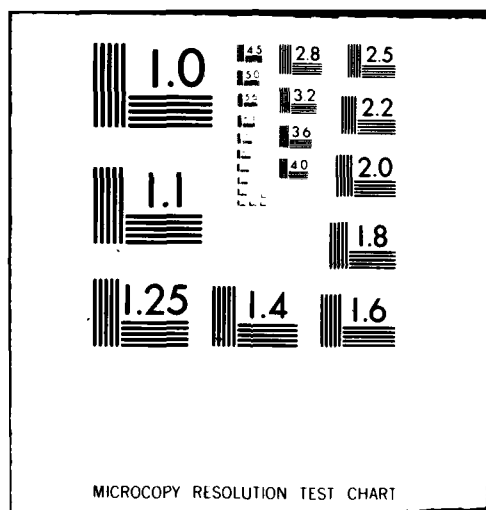
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# **FATIGUE BEHAVIOR OF ADHESIVELY BONDED JOINTS**

## **Volume II: Appendices**

*JOHN ROMANKO*

*W. G. KNAUSS*

*GENERAL DYNAMICS, FORT WORTH DIVISION  
PASADENA, CALIFORNIA*

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Final Report for Period July 1976 — December 1979

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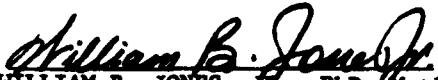
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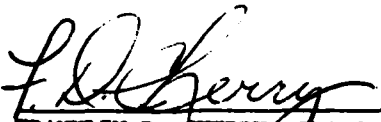
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This technical report has been reviewed and is approved for publication.

  
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The fatigue behavior of adhesively bonded aluminum joints was investigated analytically and experimentally. Stress distributions in the adhesive interlayer were calculated to guide planning of the joint tests and the interpretation of the test results. Finite element stress analyses were conducted using a linear elastic, and a three-element linear viscoelastic model for (continued)		

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the adhesive behavior. Thick adherend lap-shear specimens of the model joint configuration were fabricated and tested. These specimens were instrumented and subjected to sinusoidal fatigue tests at several frequencies, humidities and temperatures. Certain tests were interrupted at various stages and the specimens examined for fatigue damage. Damage was found to initiate in the (calculated) high stress regions, and to propagate with further load cycling. Observations and recommendations resulting from the investigation are included in this final report.

## FOREWORD

This technical report describes the research conducted for the Air Force Wright Aeronautical Laboratory, Air Force Systems Command, Wright-Patterson Air Force Base, OH 45433, under Contract F33615-76-C-5220, with Dr. W. B. Jones, Jr., MLBC, as Project Engineer.

Dr. John Romanko, GD/FWD Program Manager, acknowledges the interest and guidance of Dr. Jones and consultant Dr. W. G. Knauss of California Institute of Technology in this program. Many scientists of Materials and Structures Technology, General Dynamics' Fort Worth Division, contributed to the efforts reported herein: Messrs. R. S. Chambers and L. R. Collins conducted the stress analysis task; Drs. P. L. Flynn and J. Romanko, and Messrs. R. H. McDaniel, F. C. Nordquist, M. A. Flanders and J. D. Reynolds were responsible for the materials characterization; Messrs. R. L. Jones and F. C. Nordquist conducted the cyclic fatigue testing; Ms. C. L. Amerson and Mr. M. E. Tohlen performed data evaluation, and Messrs. B. O. McCauley and E. W. Turns prepared the model joints and neat adhesive coupons, with assistance from Mr. W. L. Toothaker of the Quality Assurance Group.

This report is written in two volumes. Volume I contains the essential details of the program with Volume II contributing the supportive data and derivations in Appendix form.

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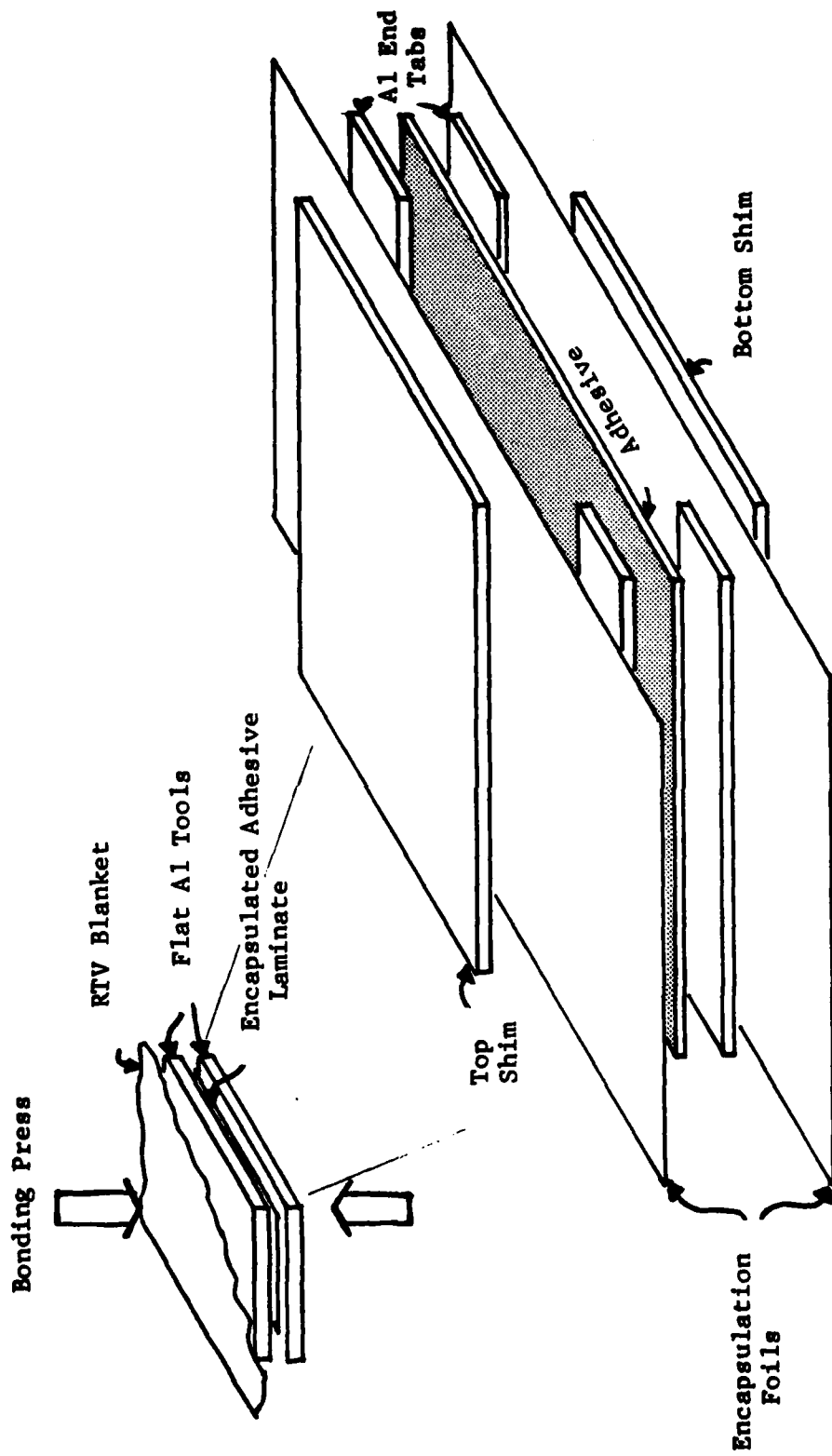


Figure A1 Cure Lay-Up for Adhesive Characterization Specimens

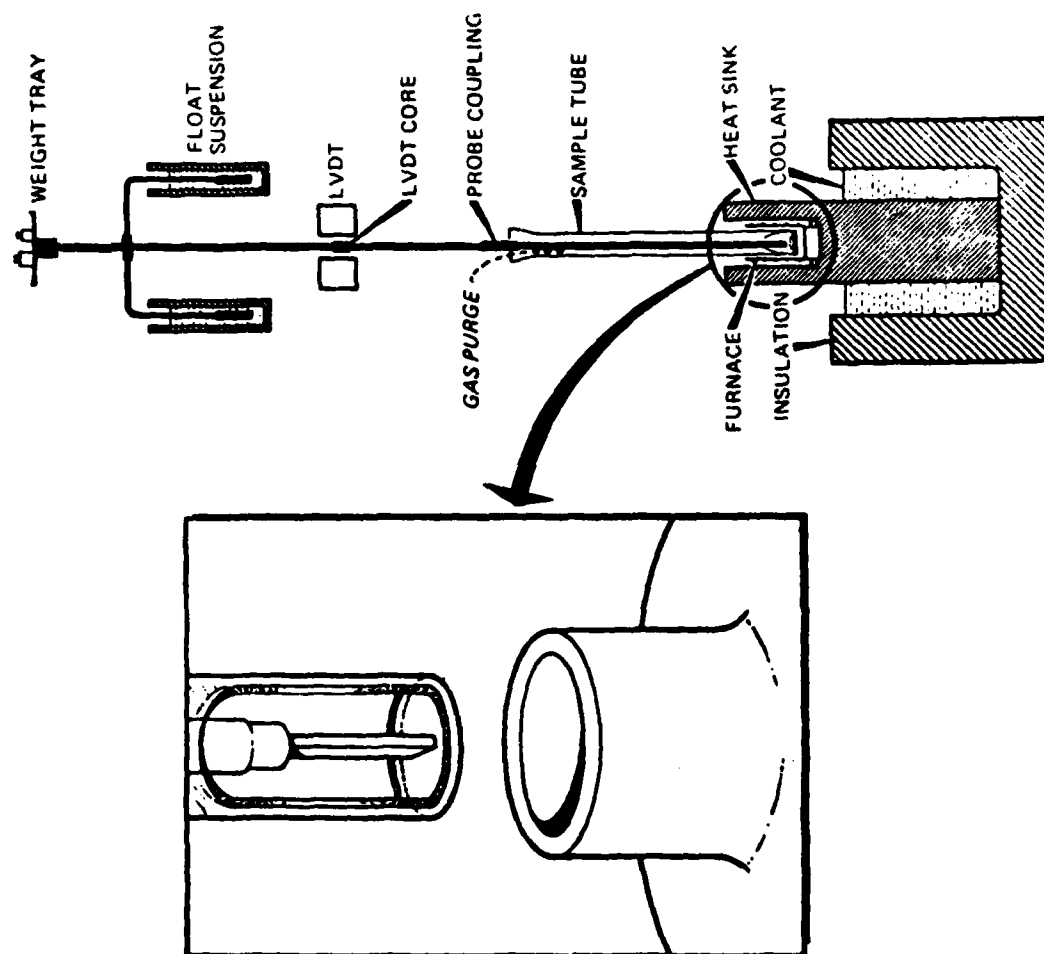
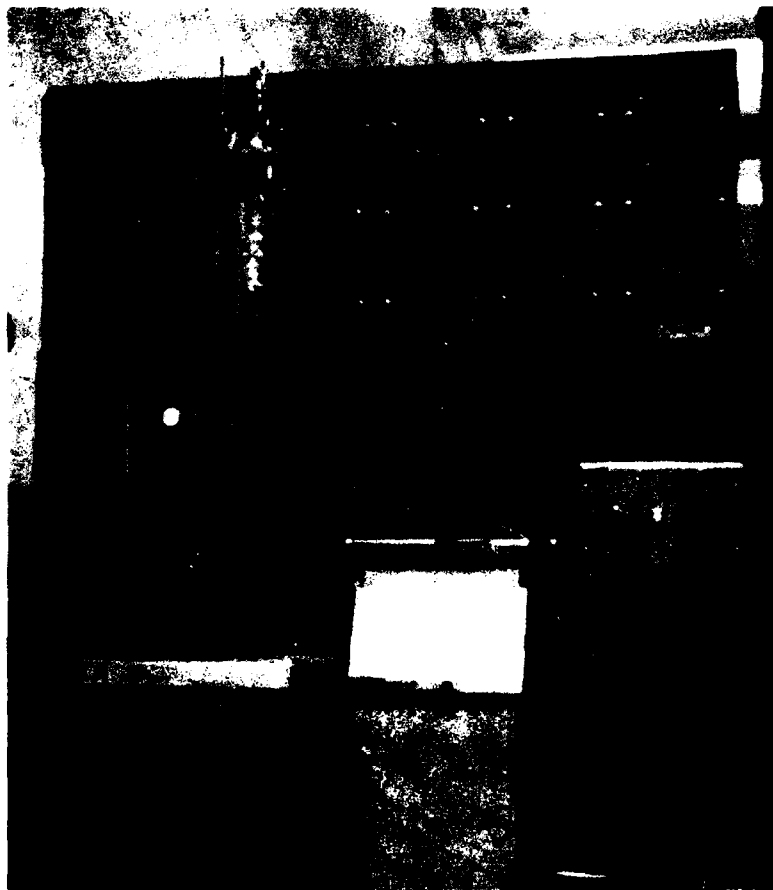


Figure A2 TMA Apparatus for Thermal Expansion Measurements (Schematic)



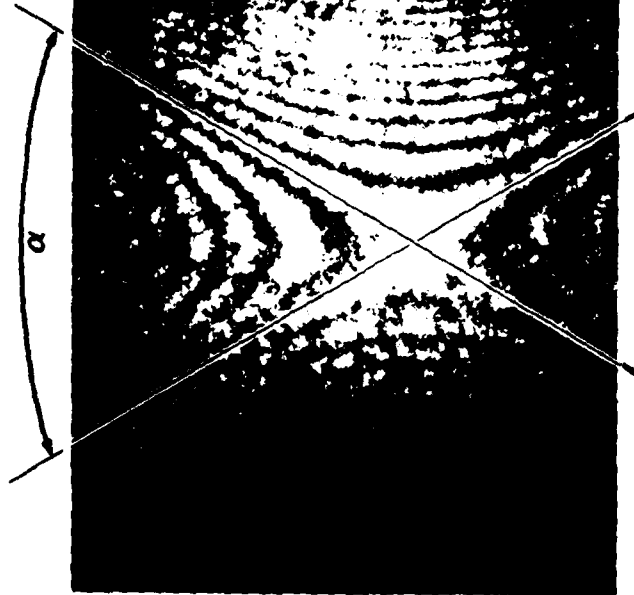
**Figure A3    Adhesive Materials Characterization Creep Frame**



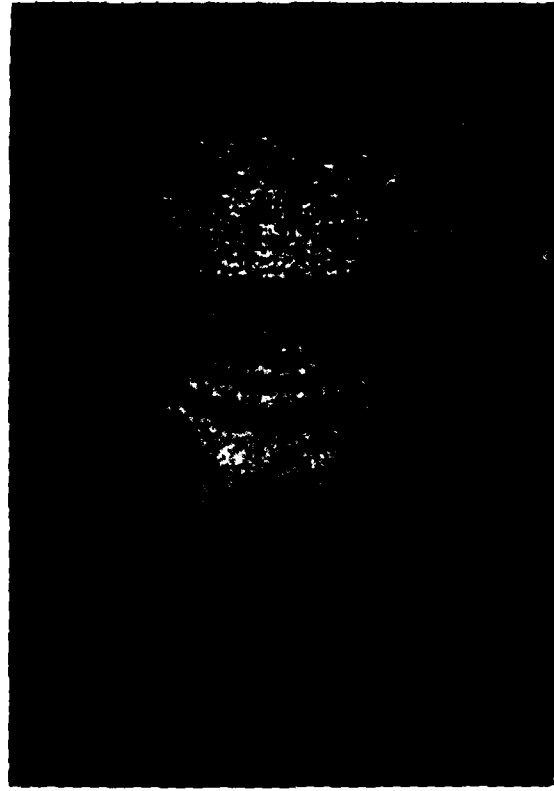


$$\nu_o = \tan^2 \frac{\alpha}{2}$$

Poisson's Ratio



(a) Holographic Fringes for 2024 T6 Al with Asymptotic Tangent Construction



(b) Holographic Fringes for FM-73M (dry) with Calculated Fringe Fit Conjugate Hyperbolae for  $\nu = 0.320$

Figure A6 Poisson's Ratio Determination from Holographic Fringe Patterns

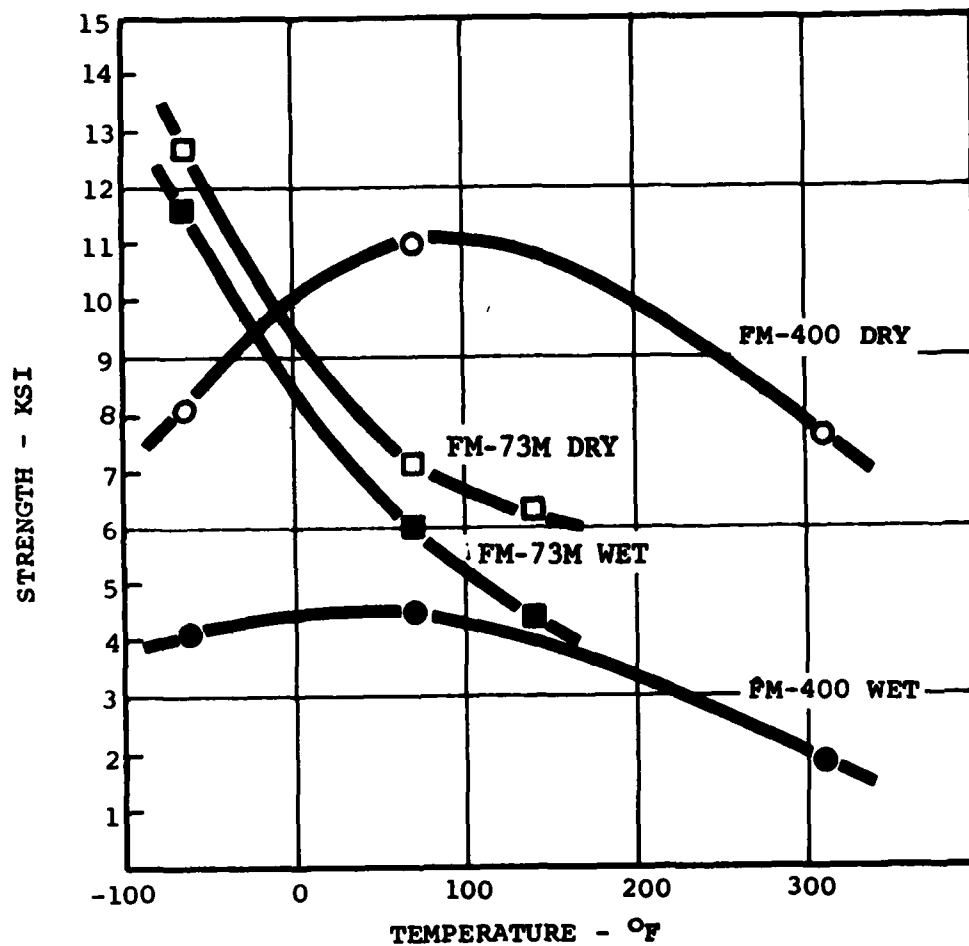


Figure A7 Strength-Temperature Histories for FM-73M and FM-400

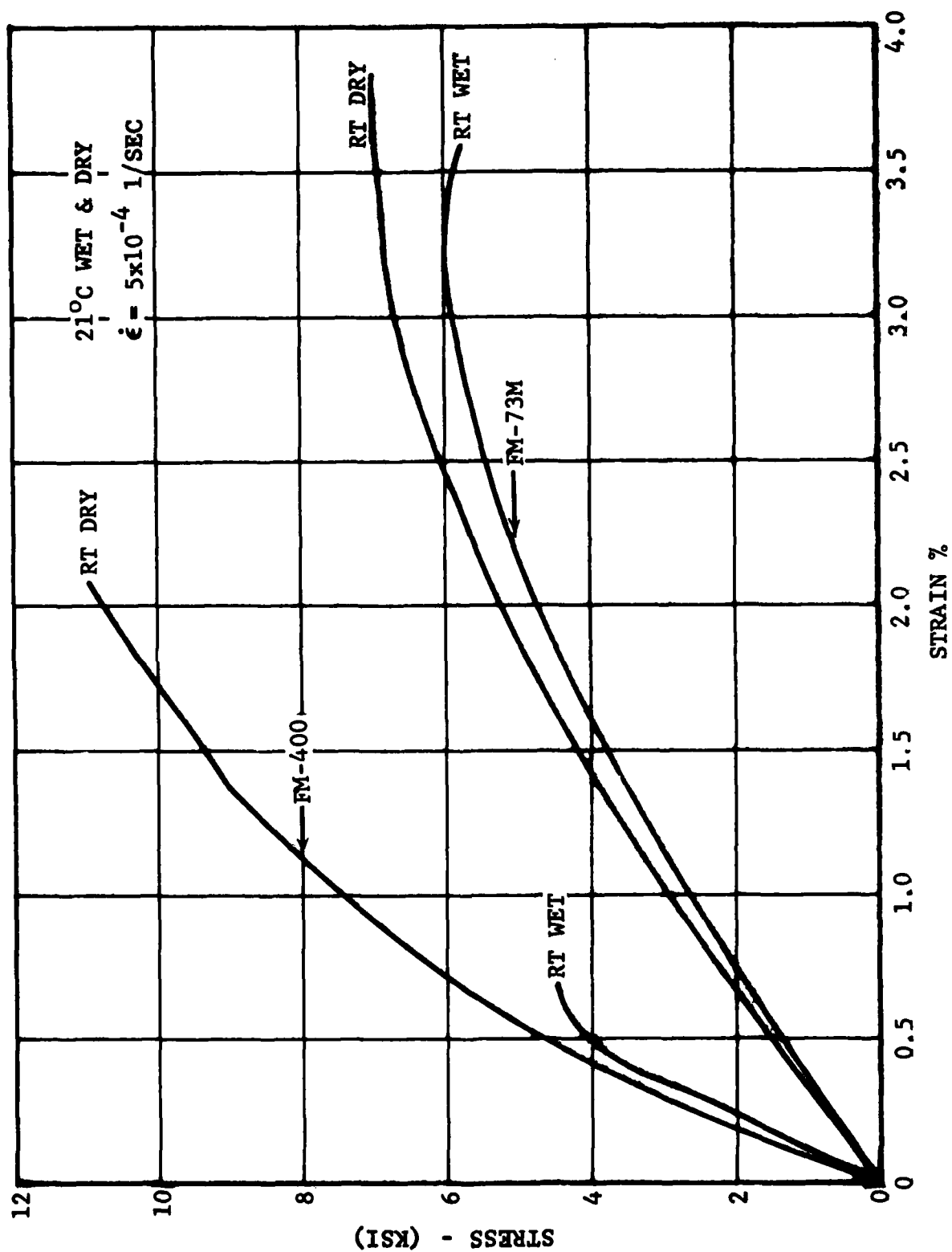


Figure A8 Stress-Strain History for FM-400

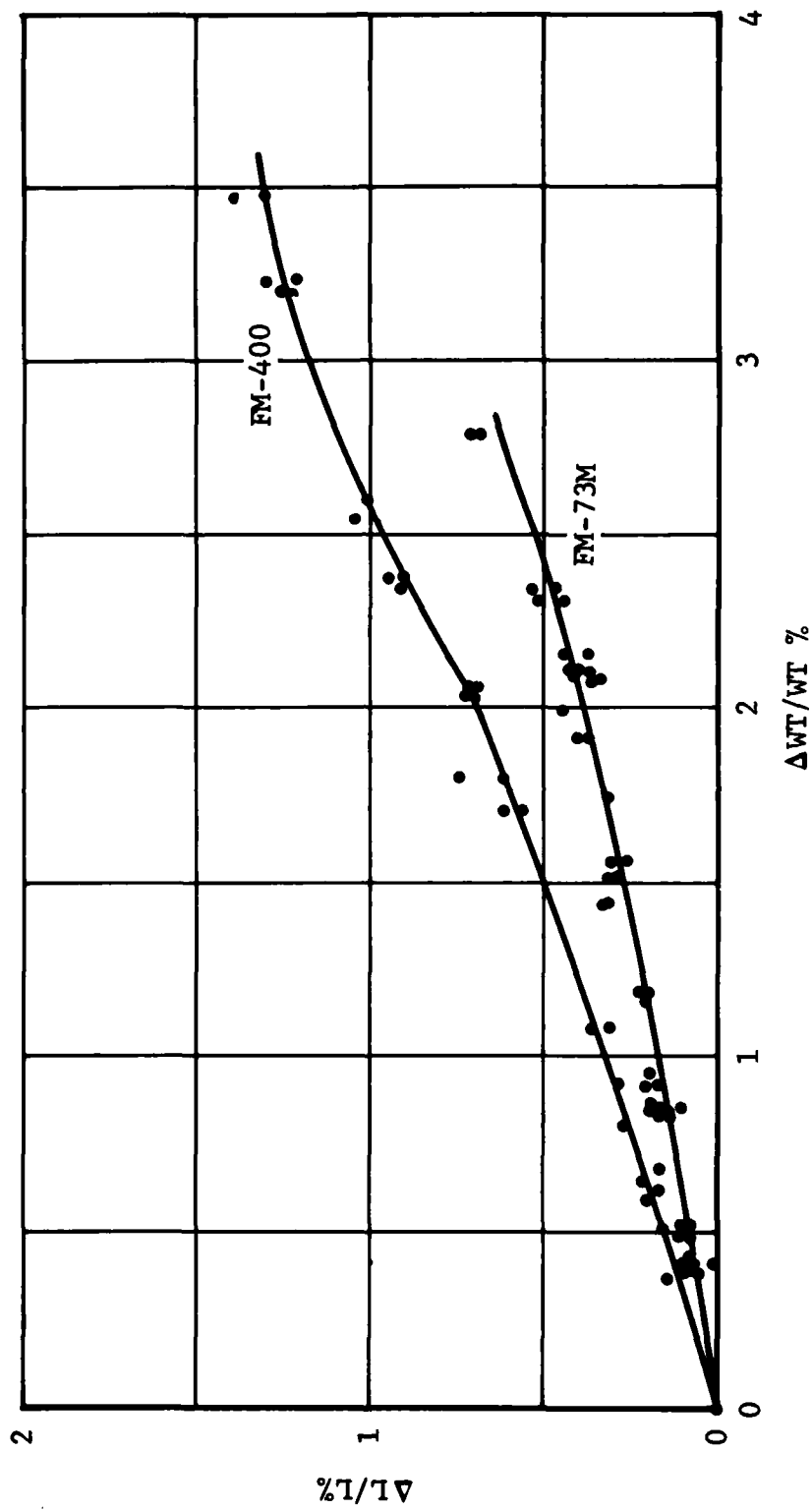
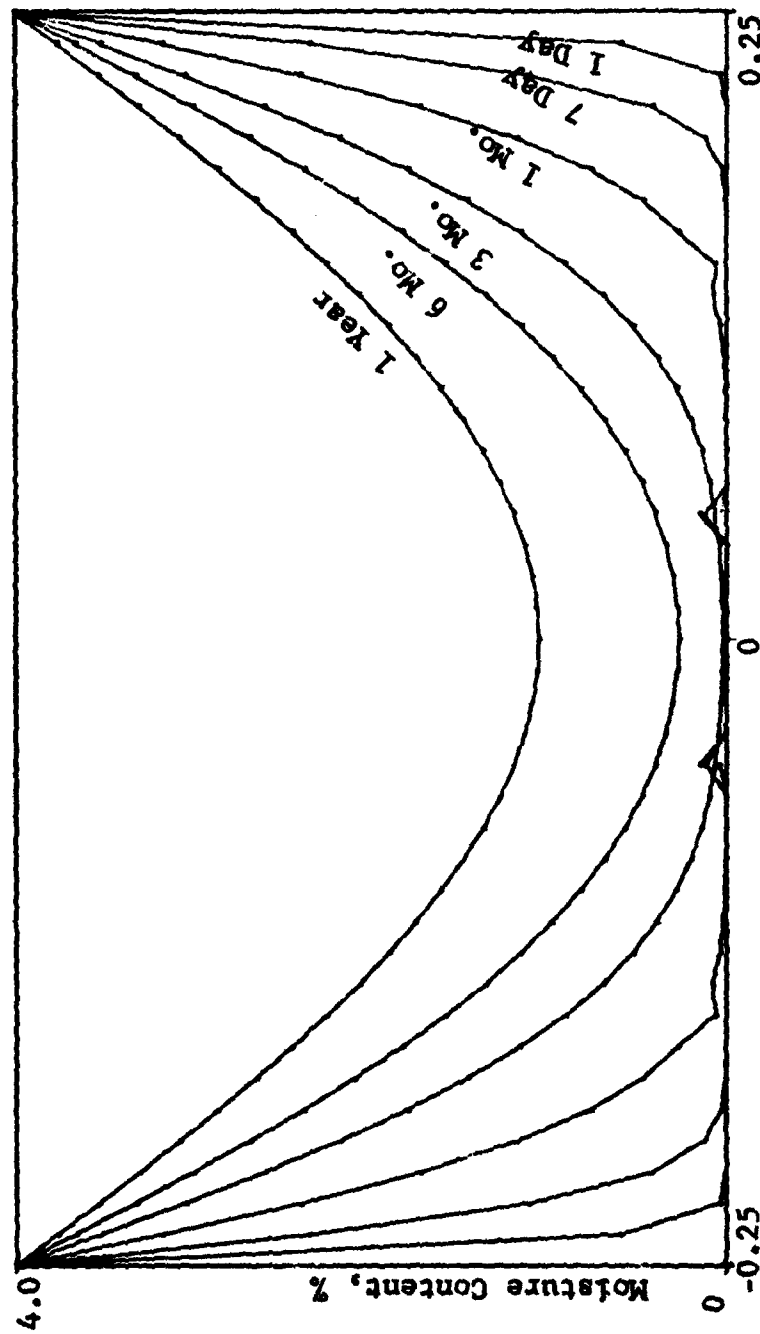


Figure A9 Moisture Expansion Curves for FM-73M and FM-400

$$D_{H2O} = 3.78 \times 10^{-5} \text{ in}^2/\text{day}, \quad P_e = 4.0\%$$



Overlap Width, Inches

Figure A10 Moisture Distribution in a 0.5-Inch Thick Slab of FM-400 Adhesive Exposed to 100% RH at 80°C

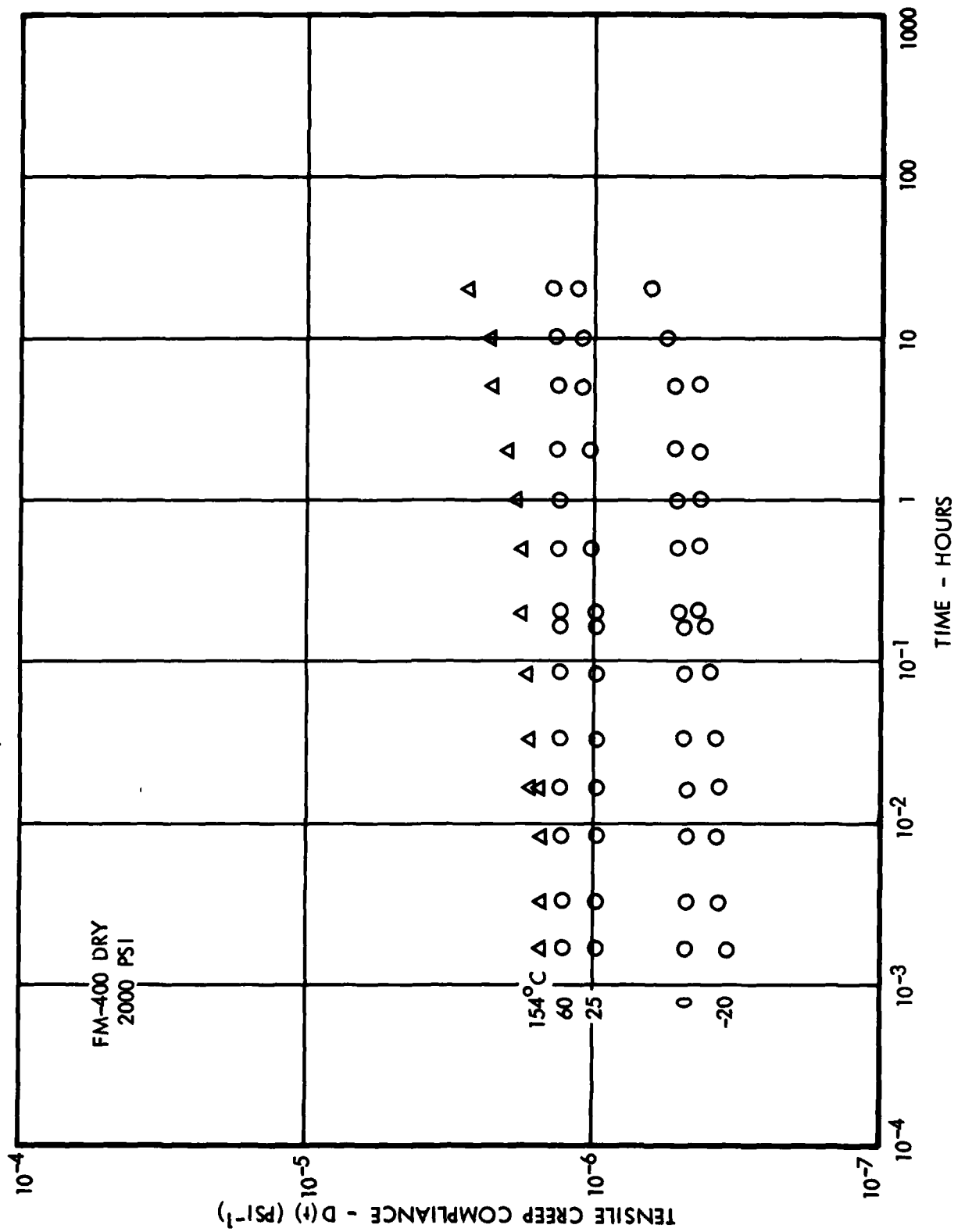


Figure A11 Tensile Creep Compliance Curves for FM-400 (dry) at Various Temperatures

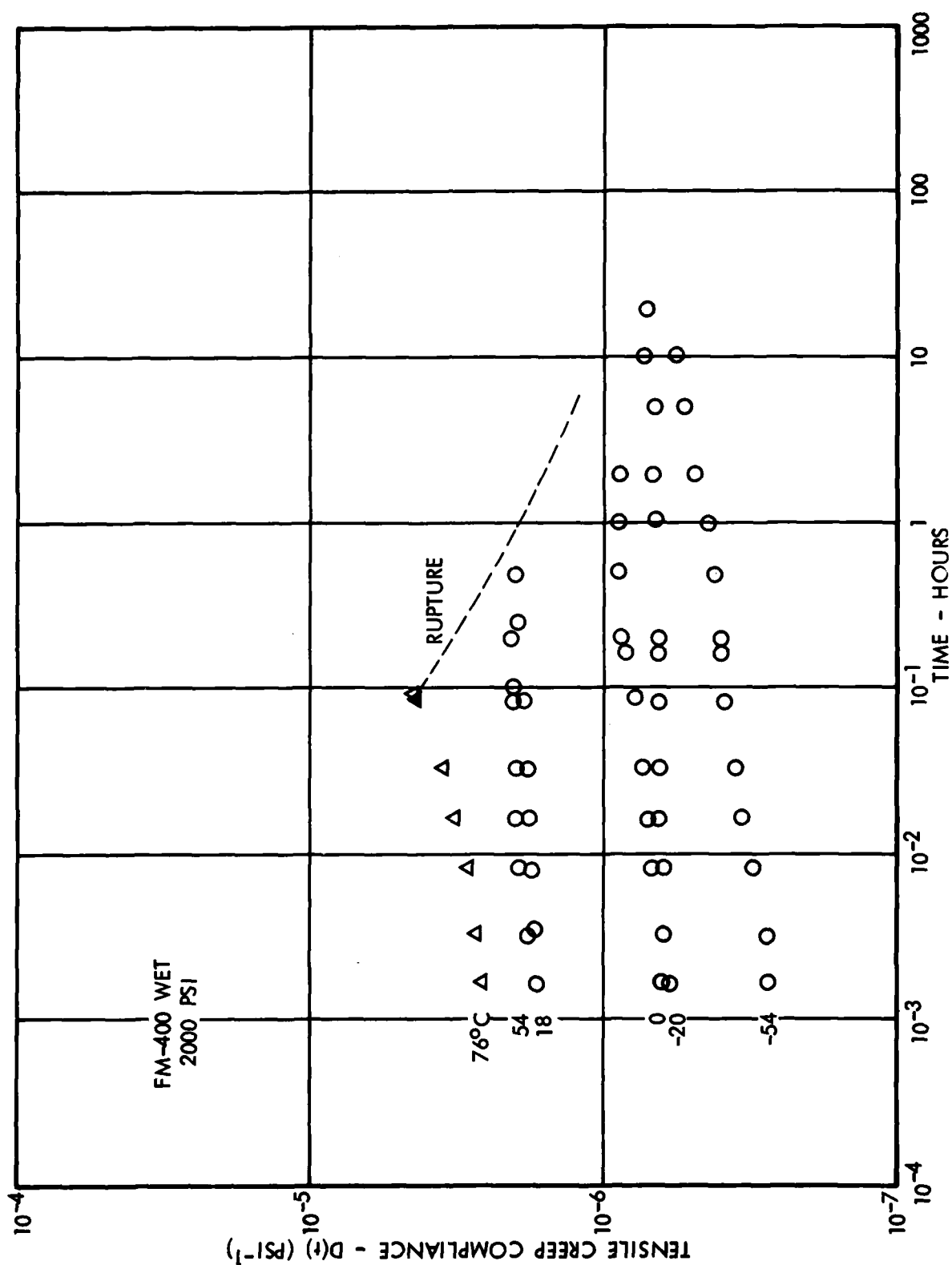


Figure A12 Tensile Creep Compliance Curves for FM-400 (wet) at Various Temperatures

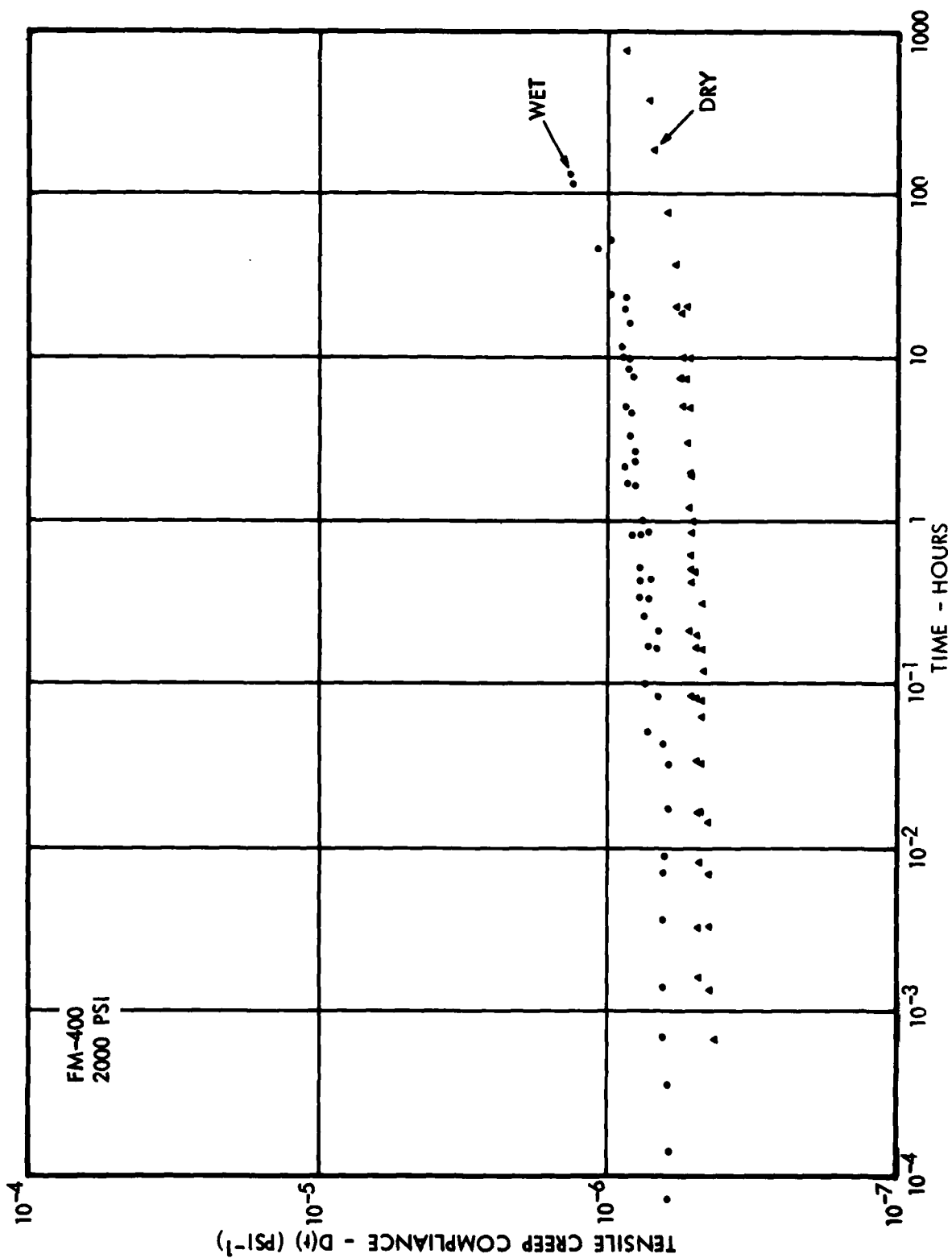


Figure A13 Tensile Creep Compliance Master Curves for FM-400 (Reduced to Reference Temperature 0°C)



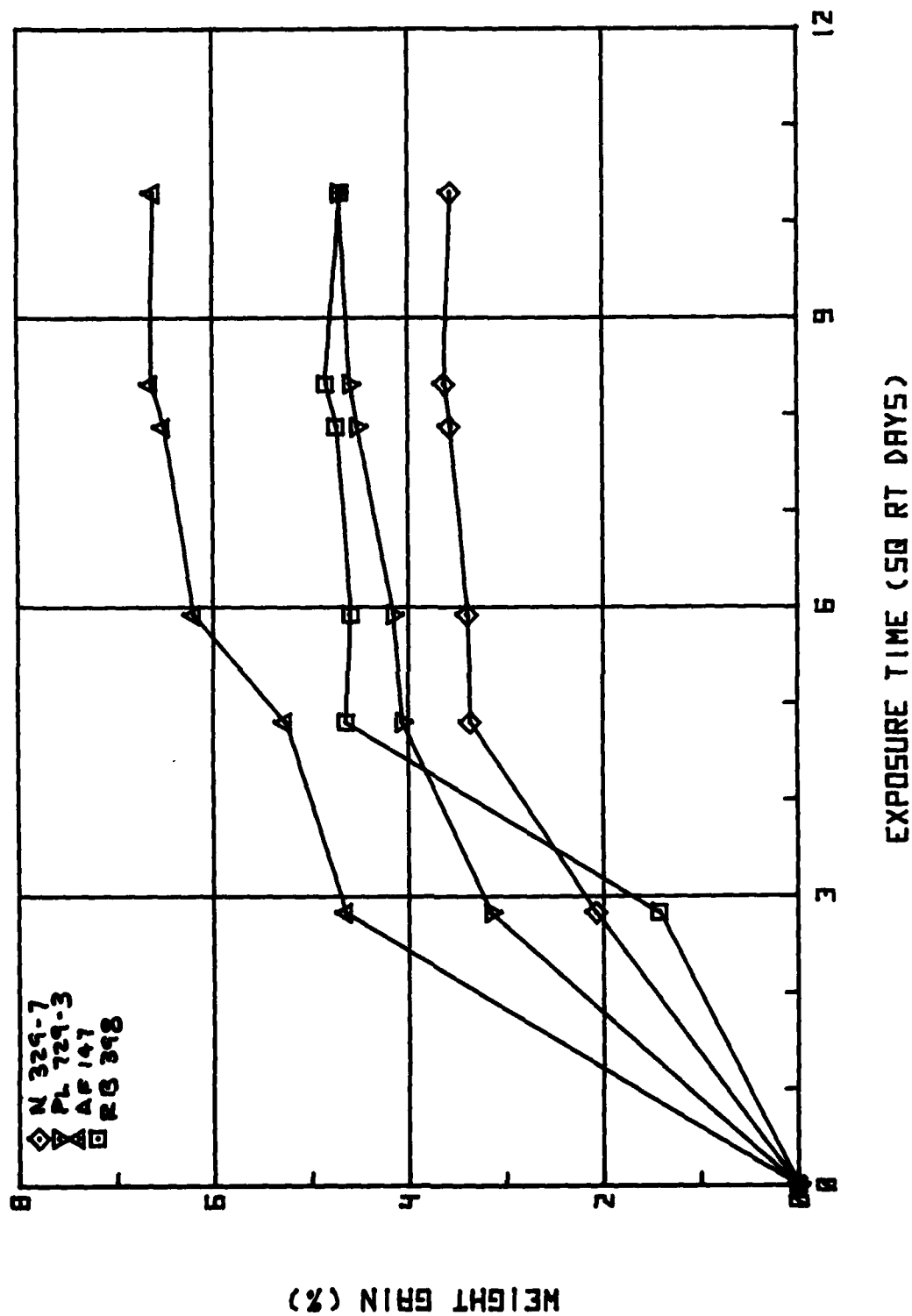


Figure A14 Weight Gain of 350°F Cured Adhesives in 150°F Water

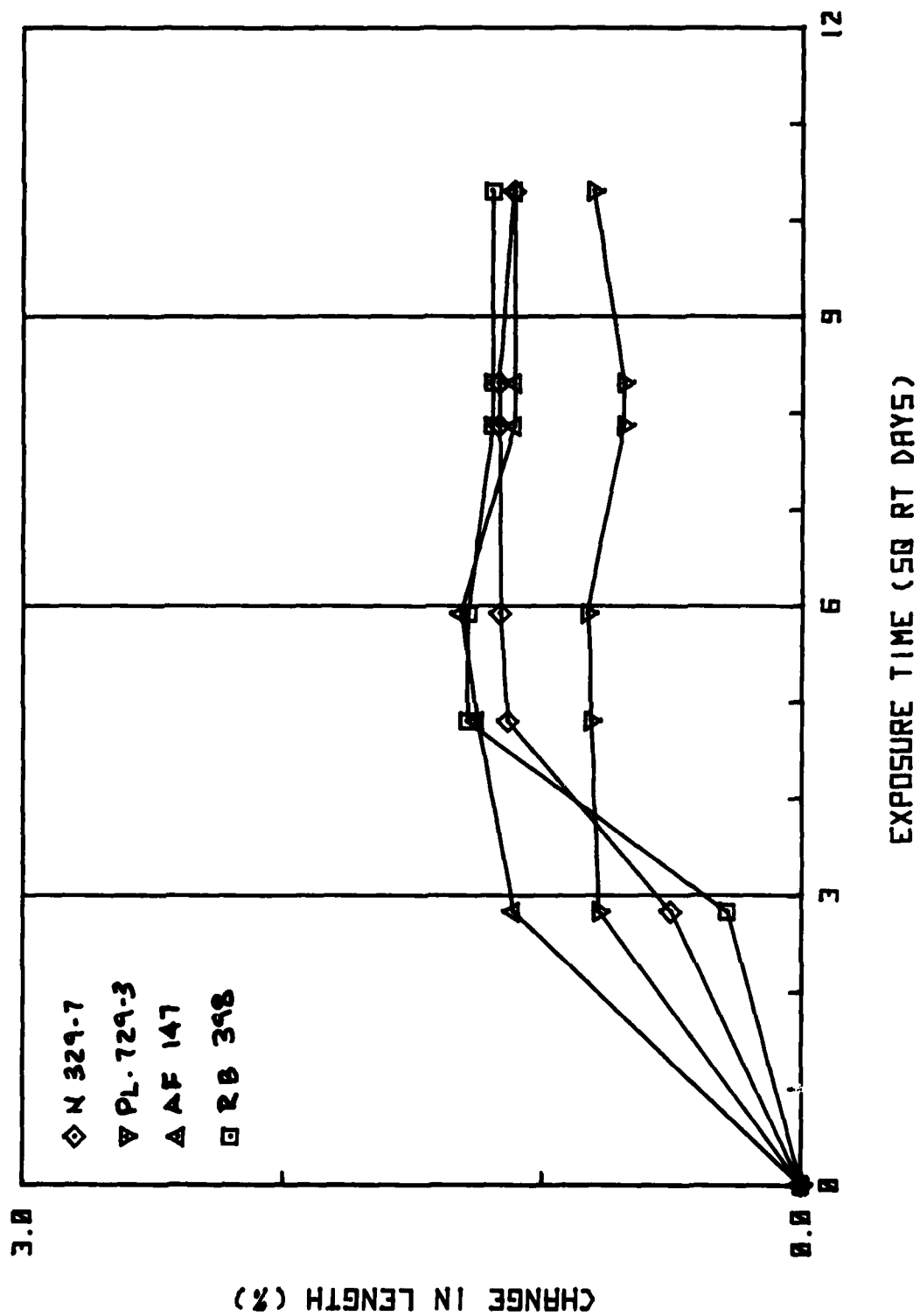


Figure A15 Length Change of 350°F Cured Adhesives in 150°F Water Vapor

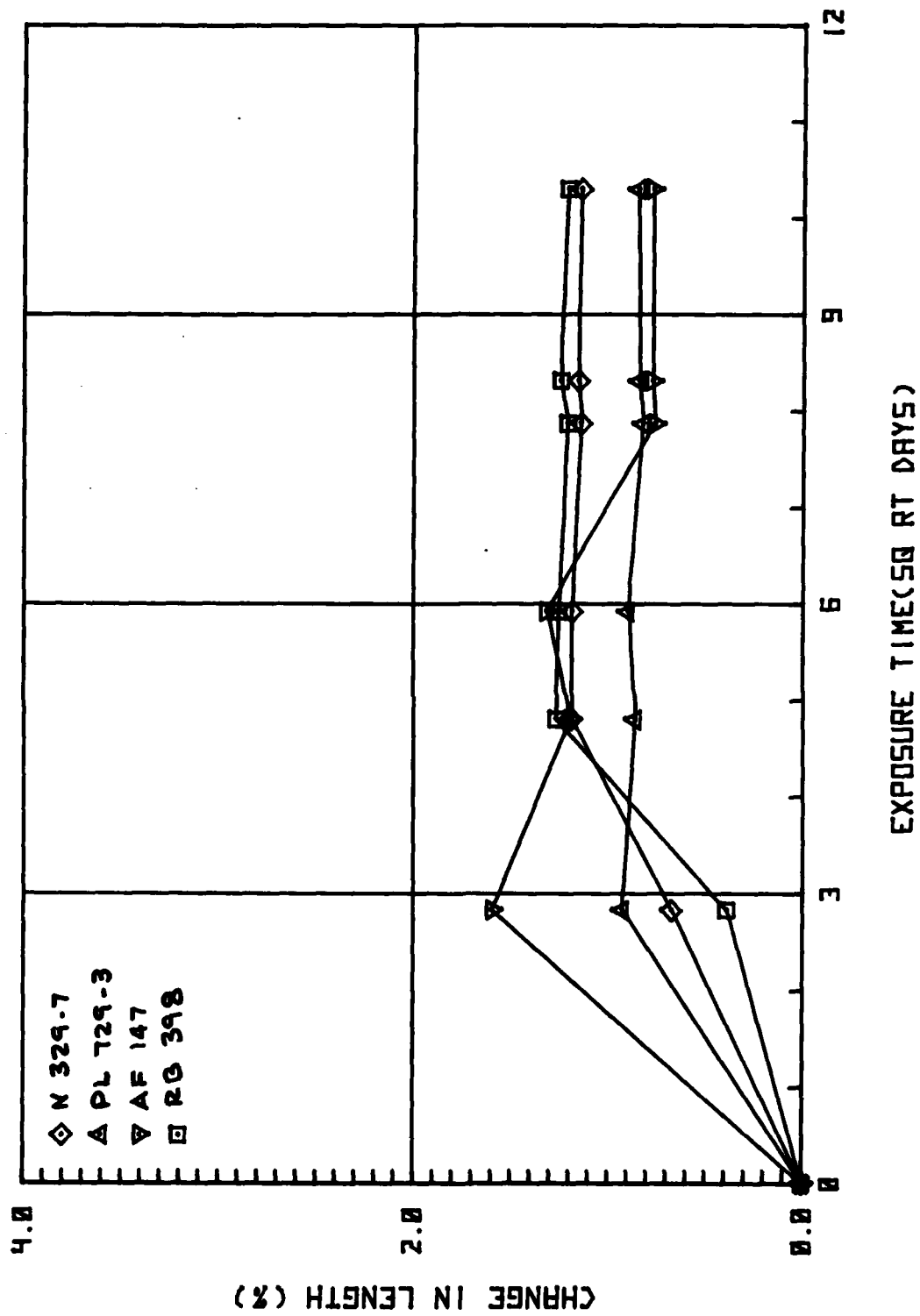


Figure A16 Length Change of 350°F Cured Adhesives in 150° Water

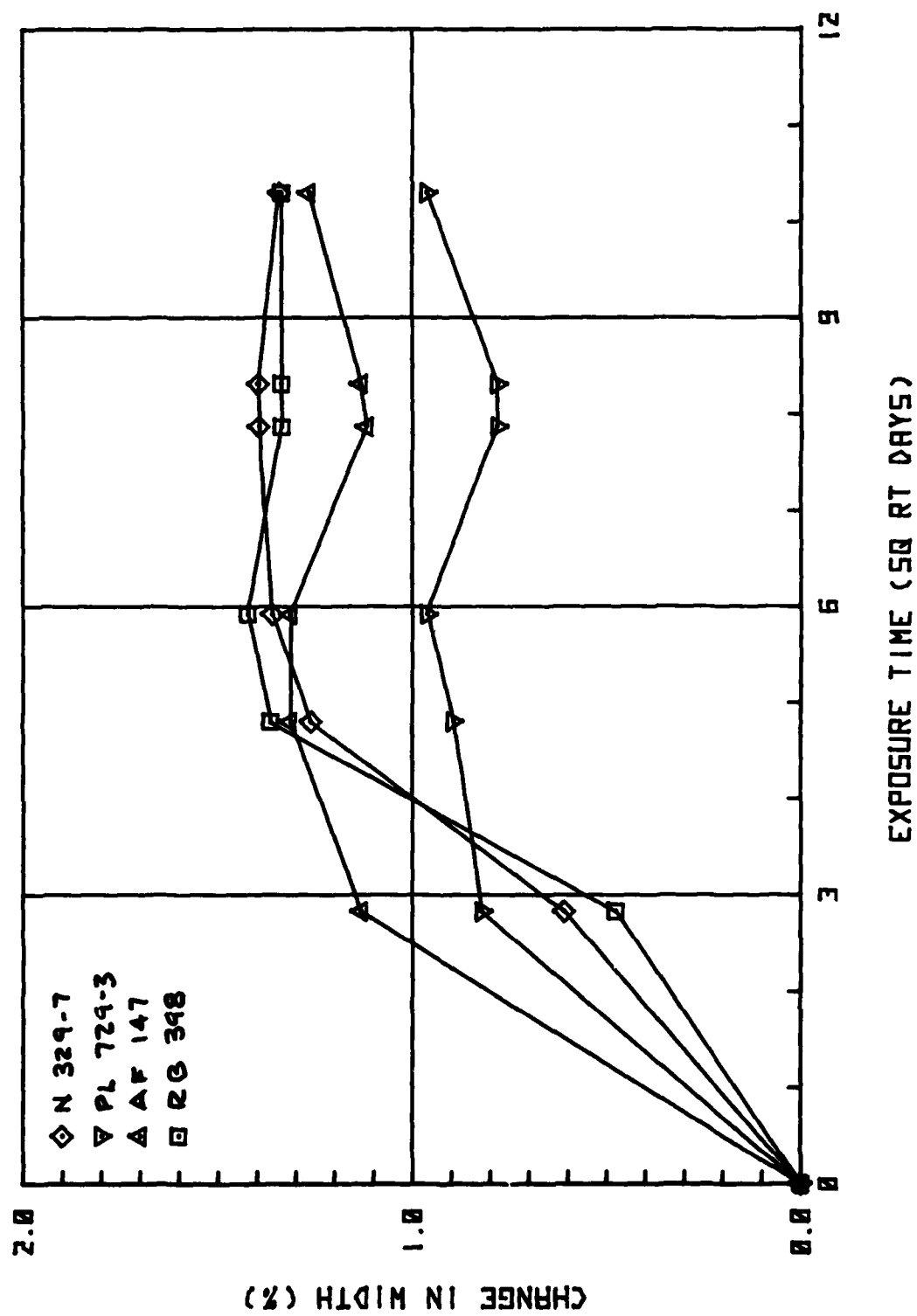


Figure A17 Width Change of 350°F Cured Adhesives in 150°F Water Vapor

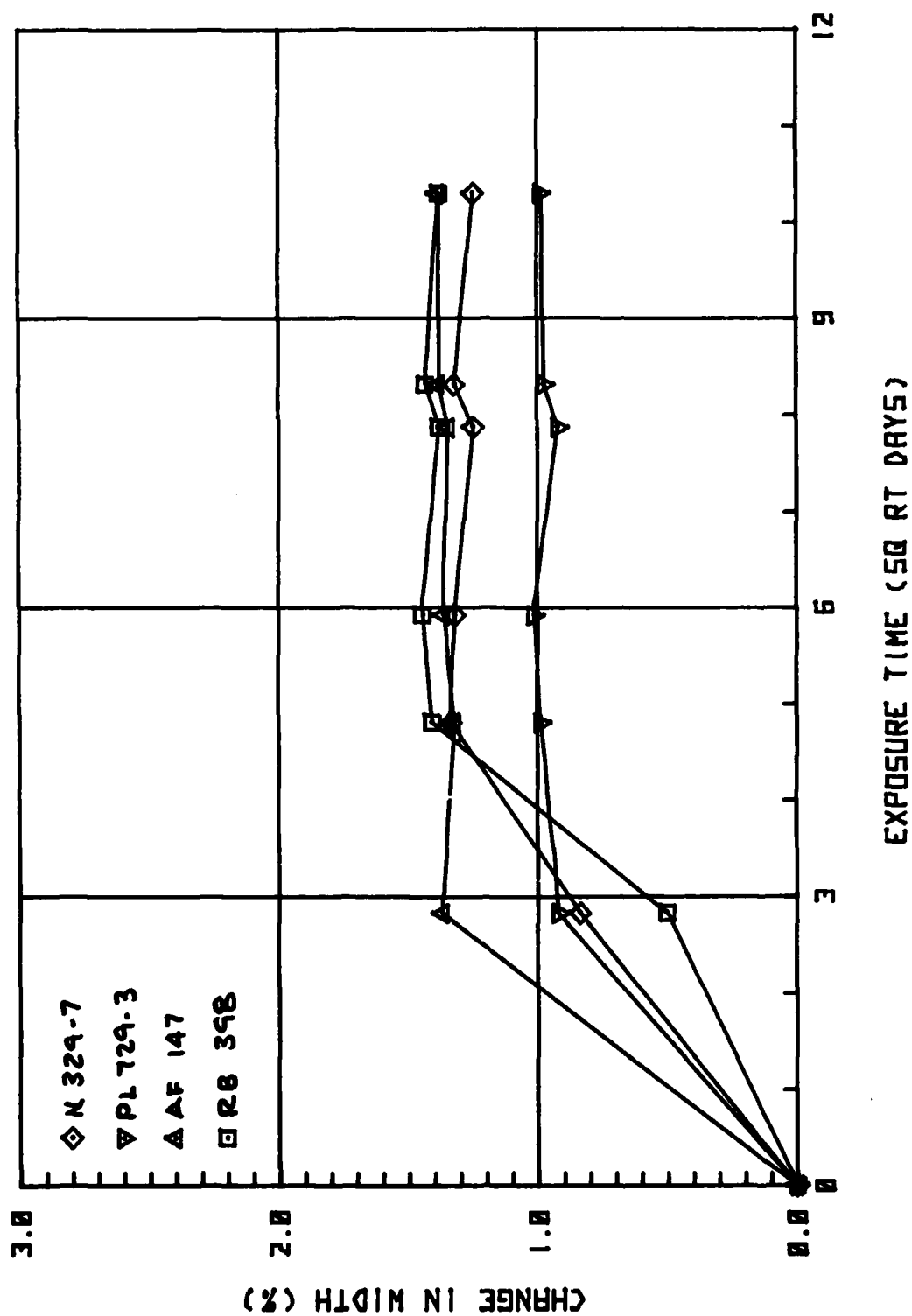


Figure A18 Width Change of 350°F Cured Adhesives in 150°F Water

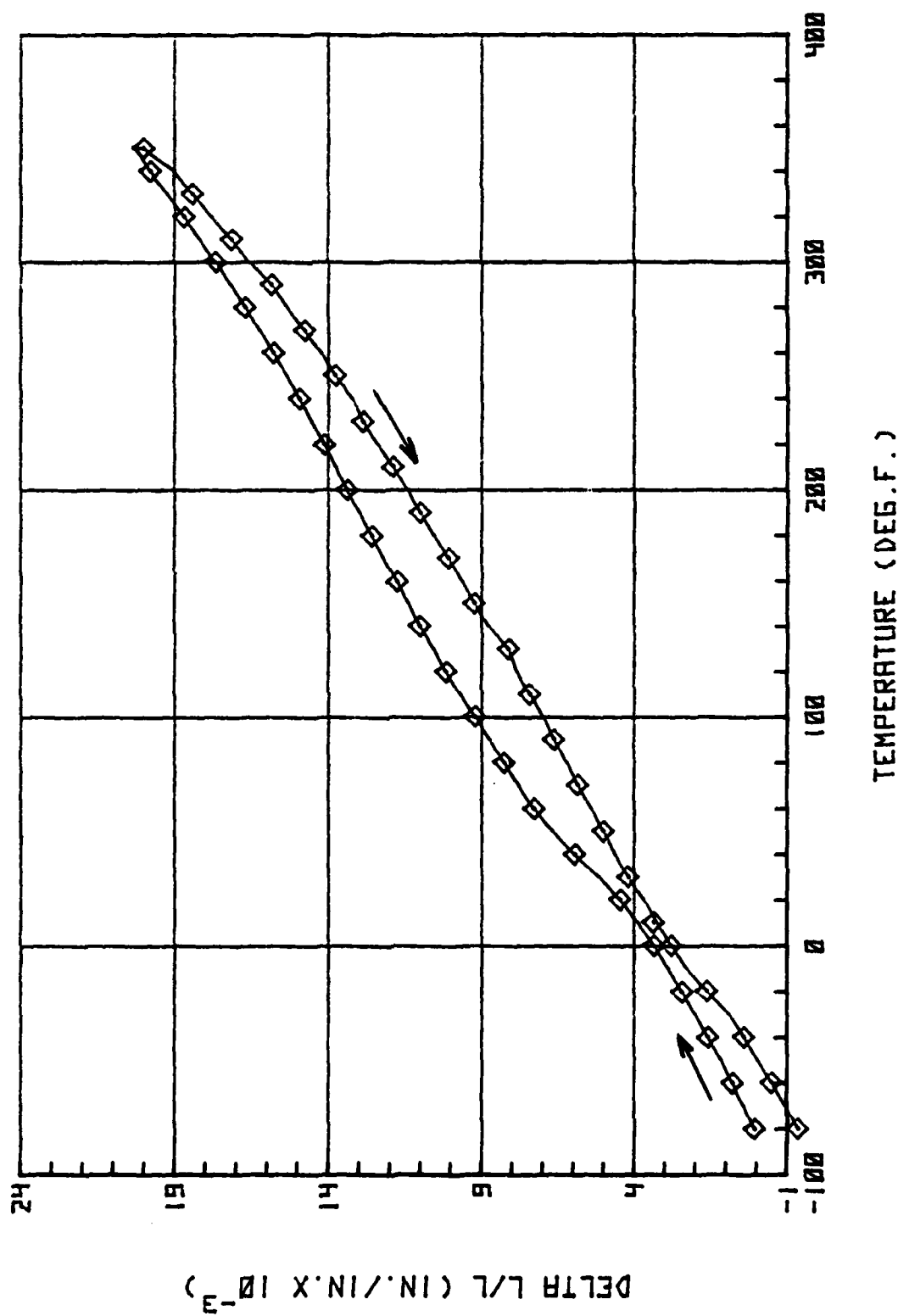


Figure A19 Thermal Expansion of Dry AF-147 Adhesive

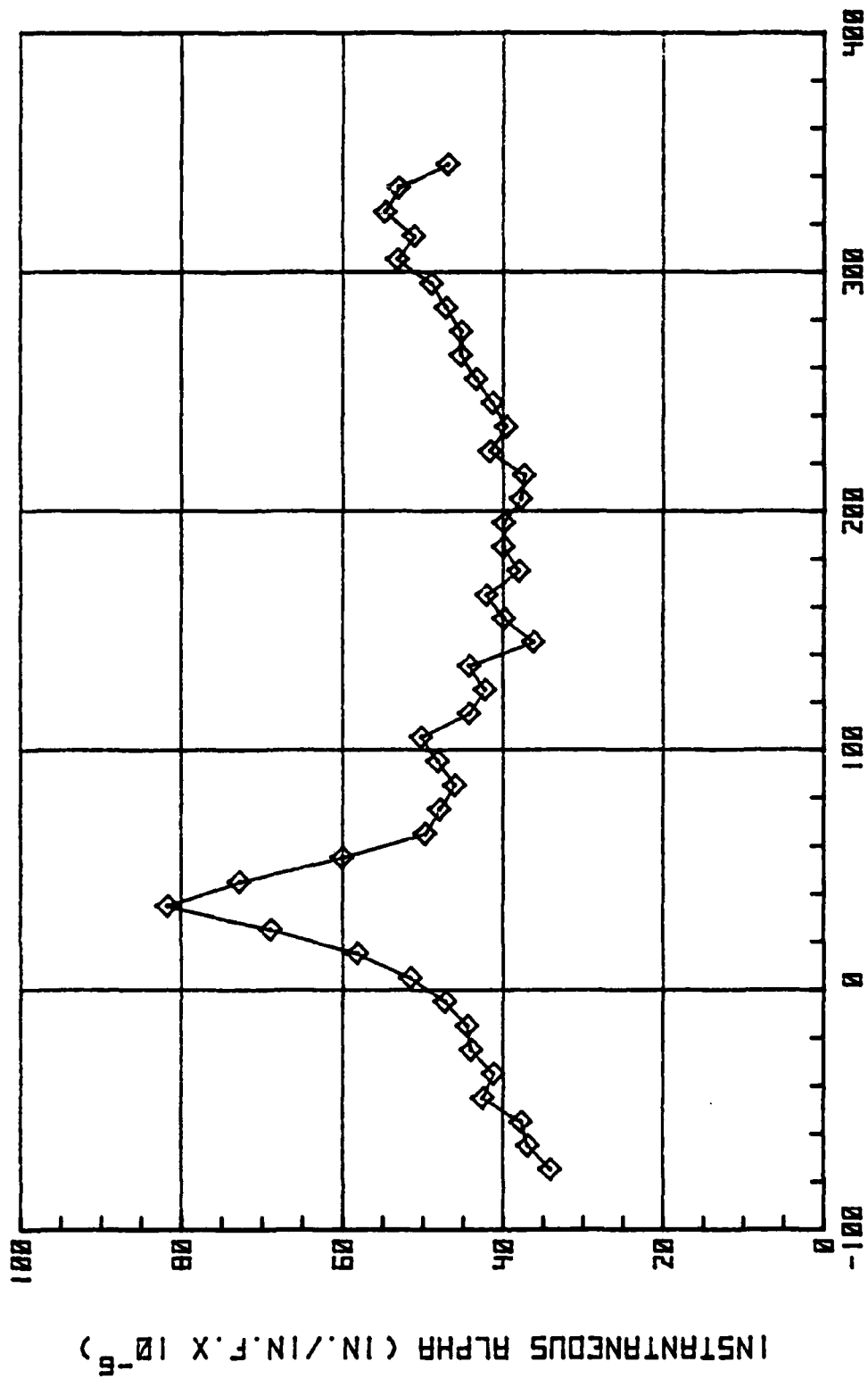


Figure A20 Thermal Expansion Coefficient of Dry AF-147

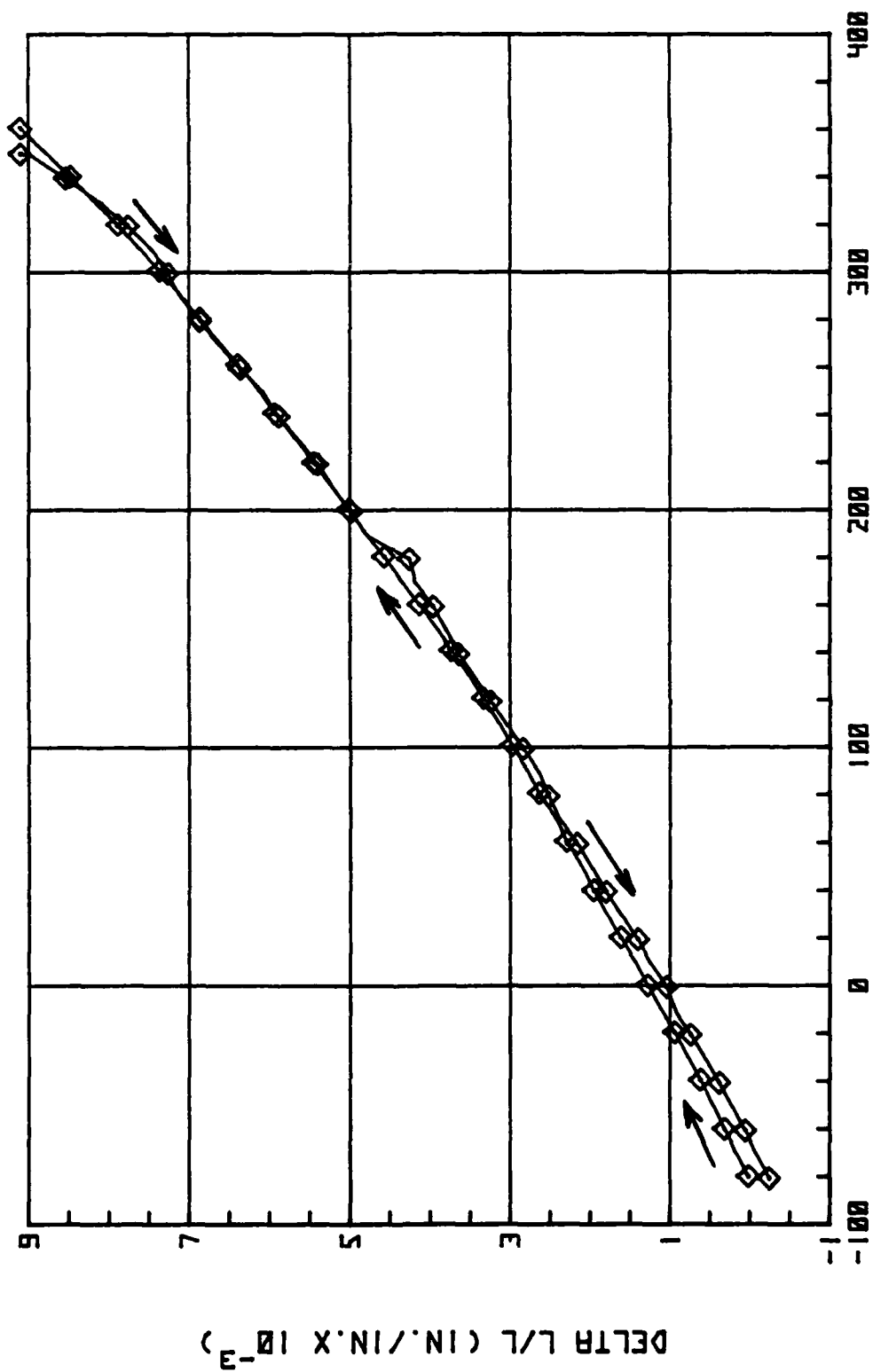


Figure A21 Thermal Expansion of Dry N329-7 Adhesive



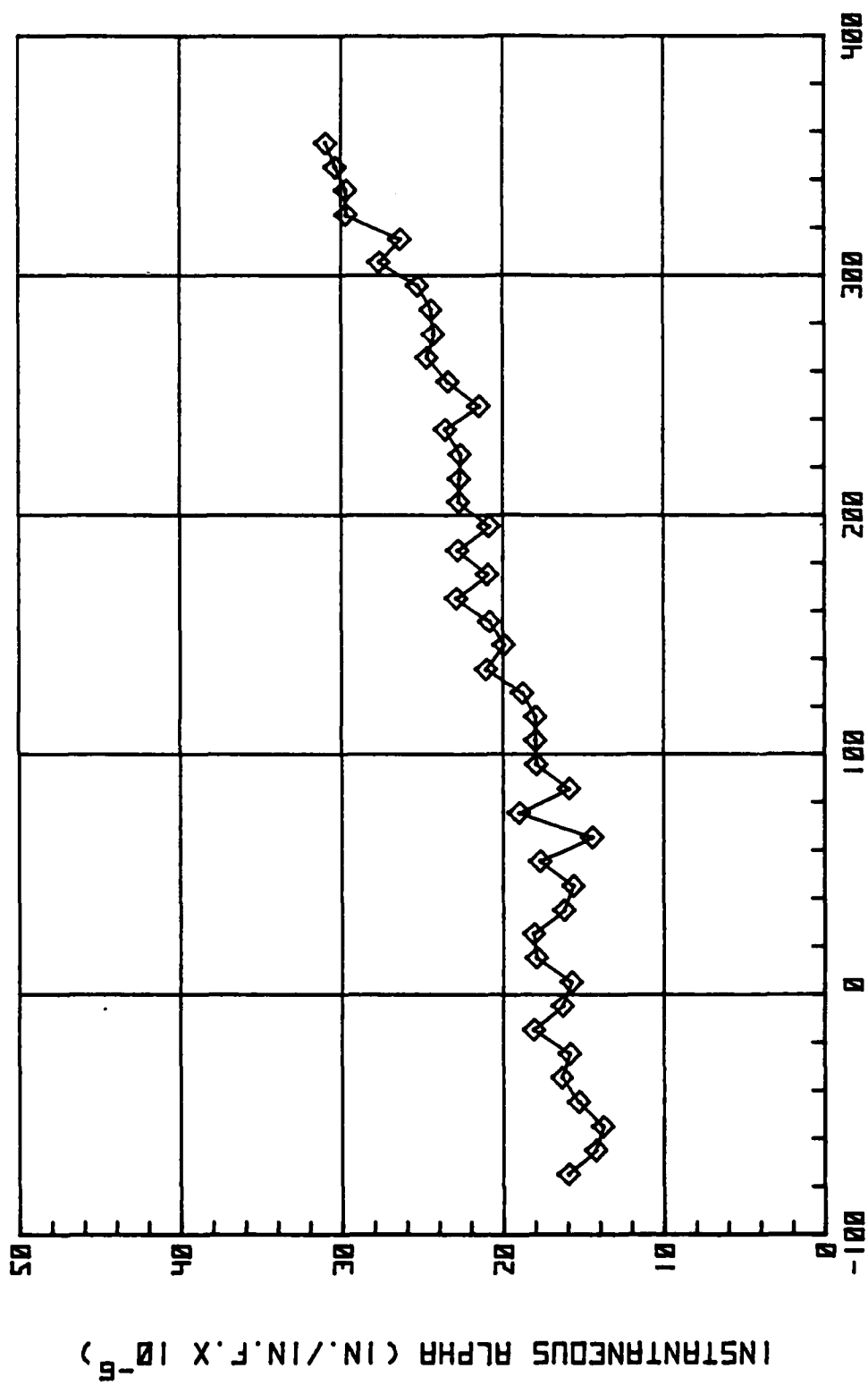
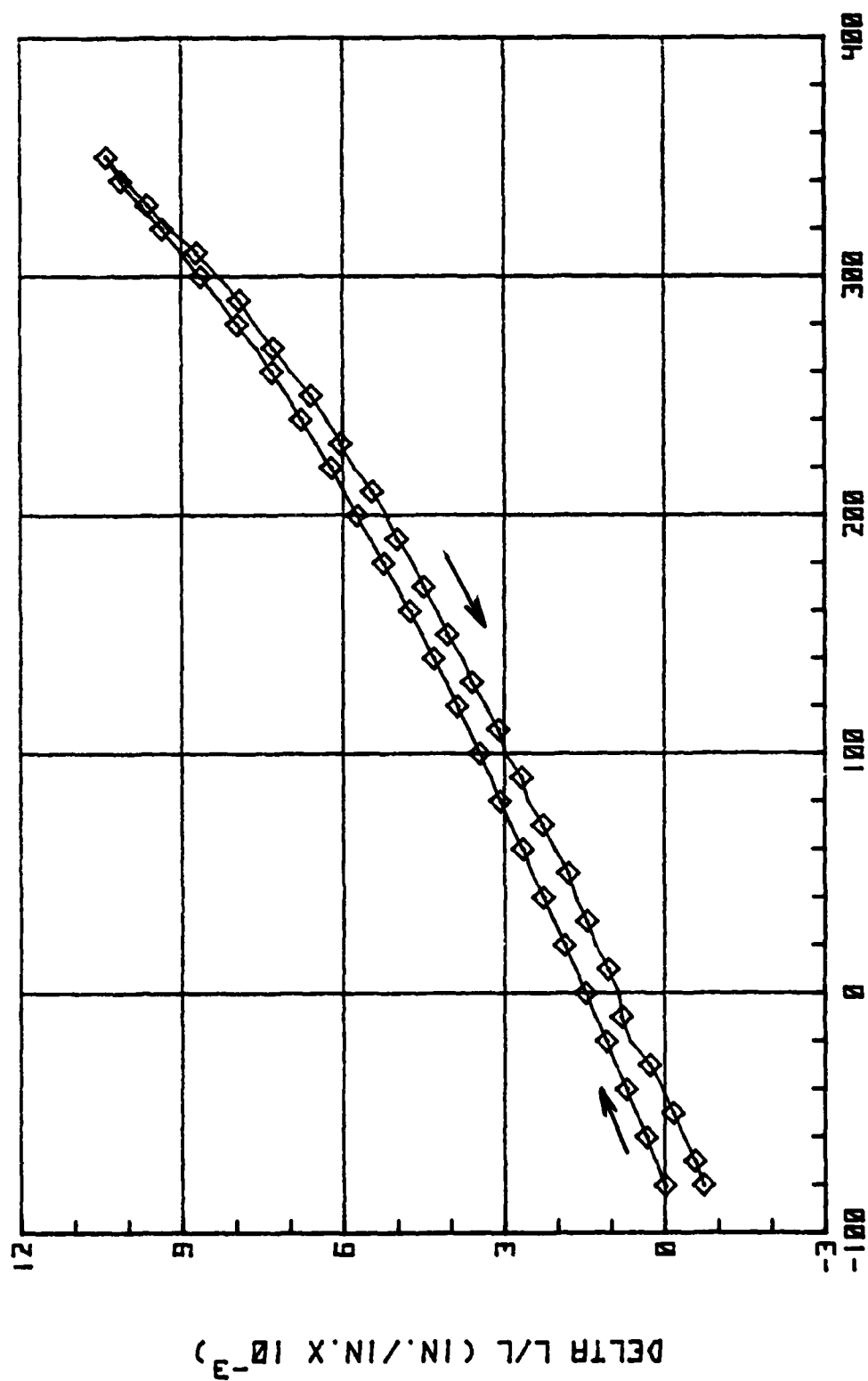


Figure A22 Thermal Expansion of Coefficient of Dry N329-7 Adhesive



TEMPERATURE (DEG. F.)

Figure A23 Thermal Expansion of Dry RB-398 Adhesive

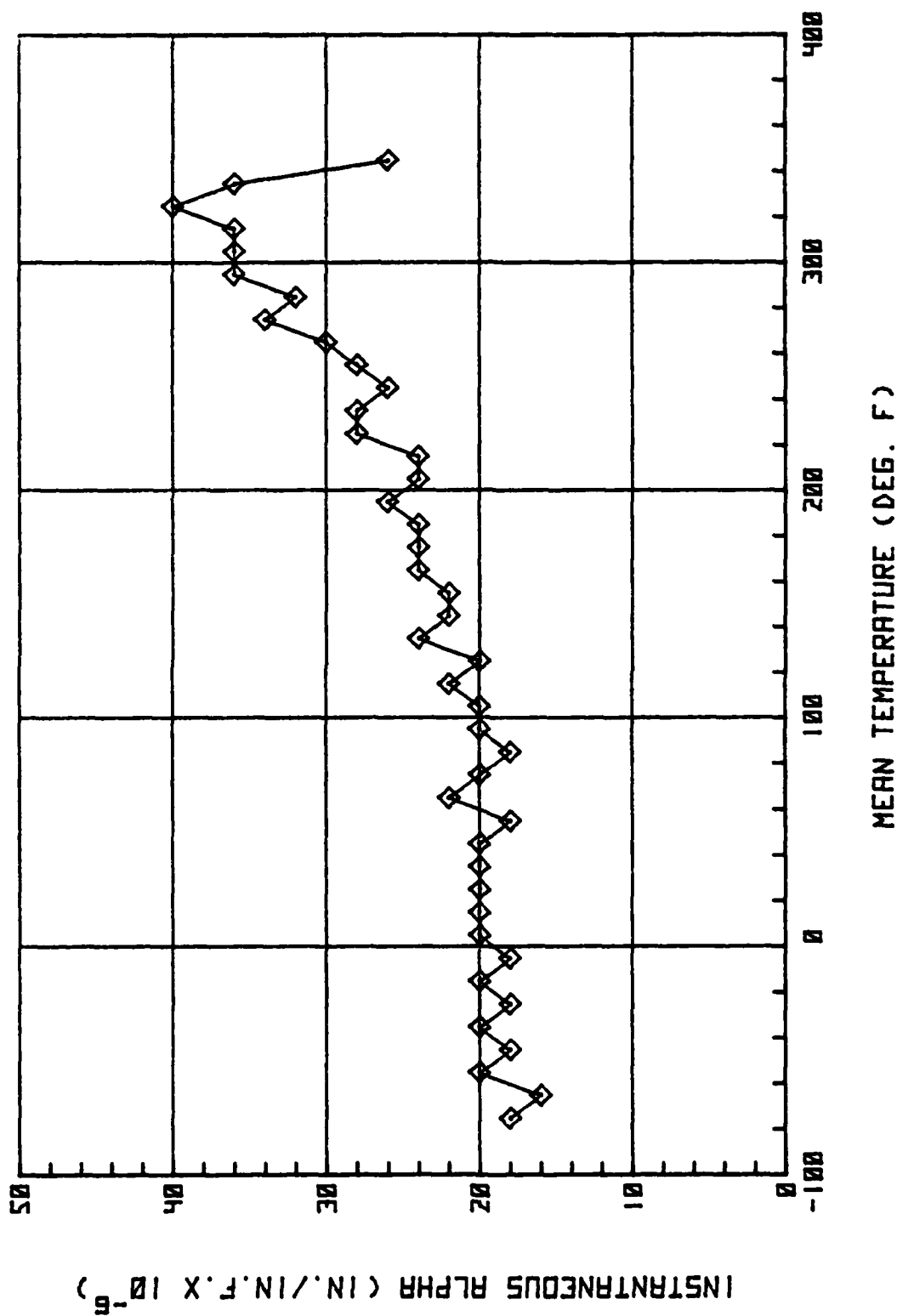


Figure A24 Thermal Expansion Coefficient of Dry RB-398 Adhesive

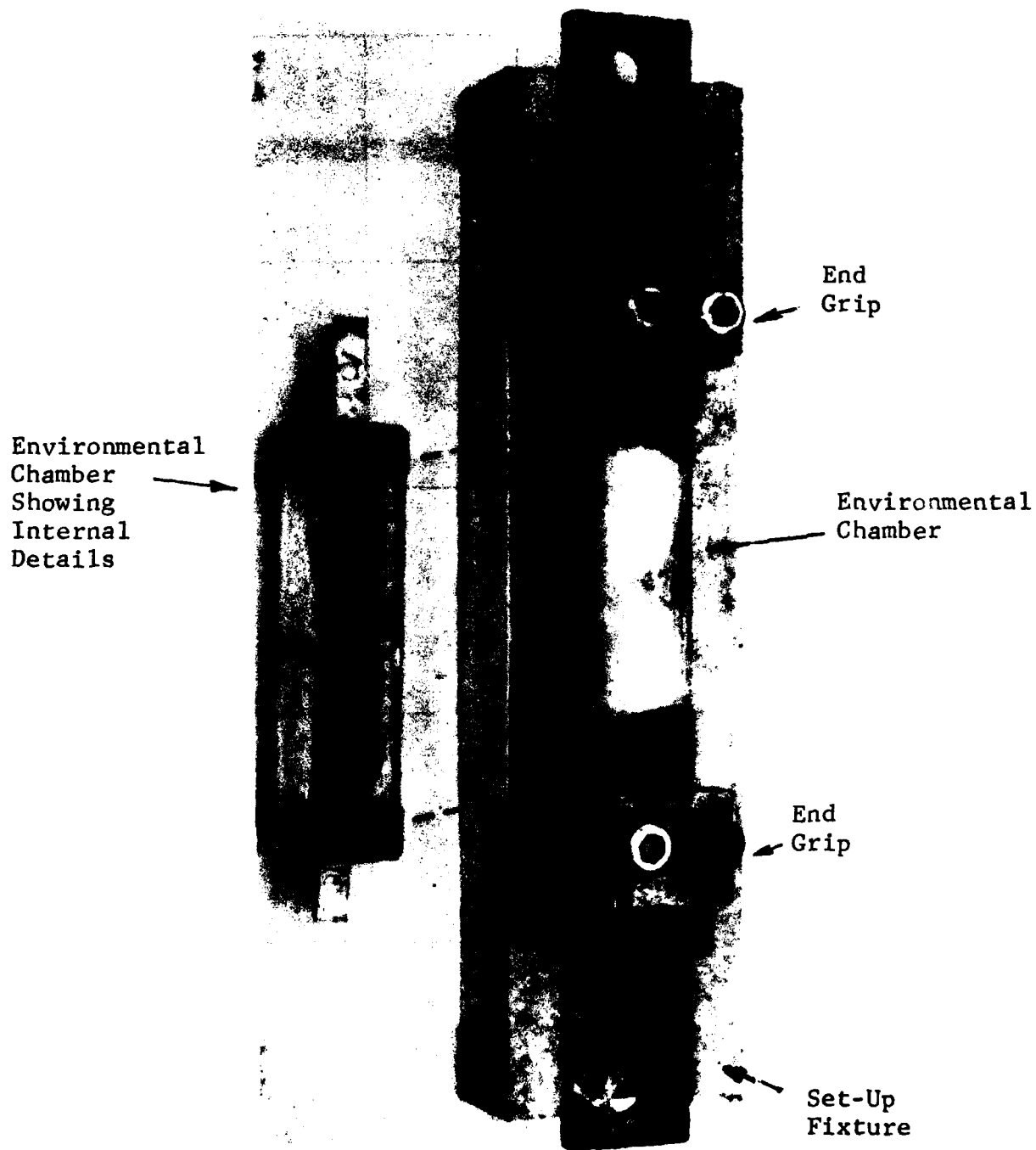


Figure A25 Environmental Chamber With Moisture-Conditioned Coupon for Smith Plot Measurements

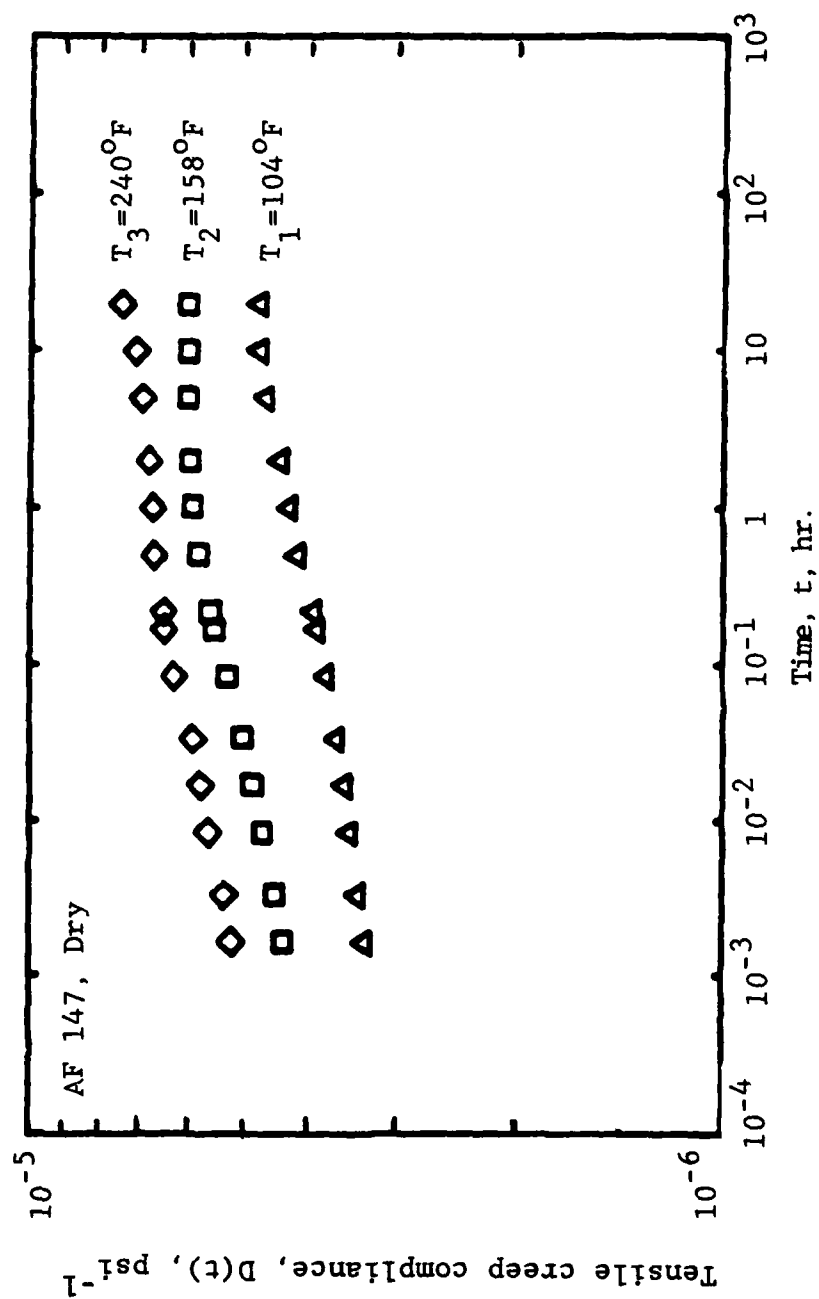


Figure A26 Tensile Creep Compliance - AF-147, Dry

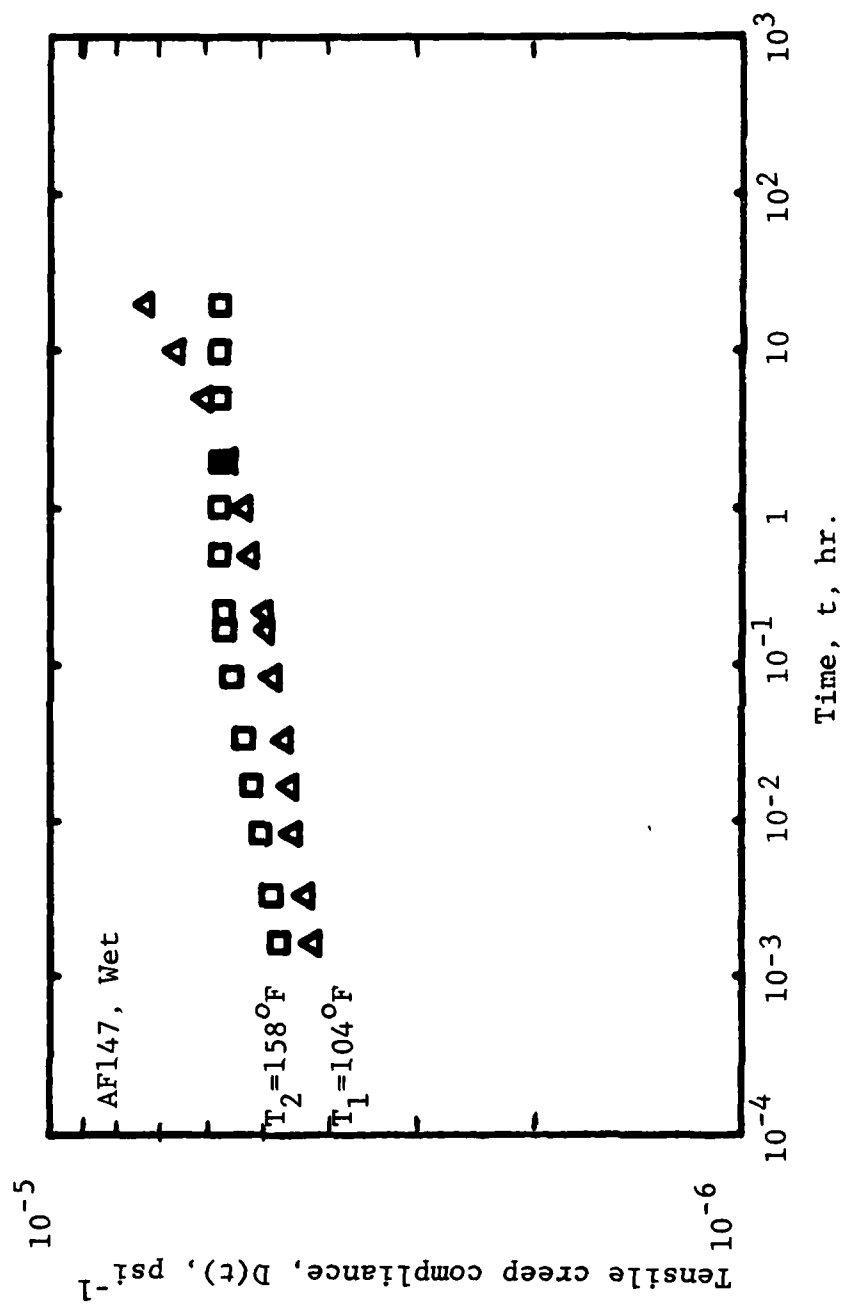


Figure A27 Tensile Creep Compliance of Wet AF-147

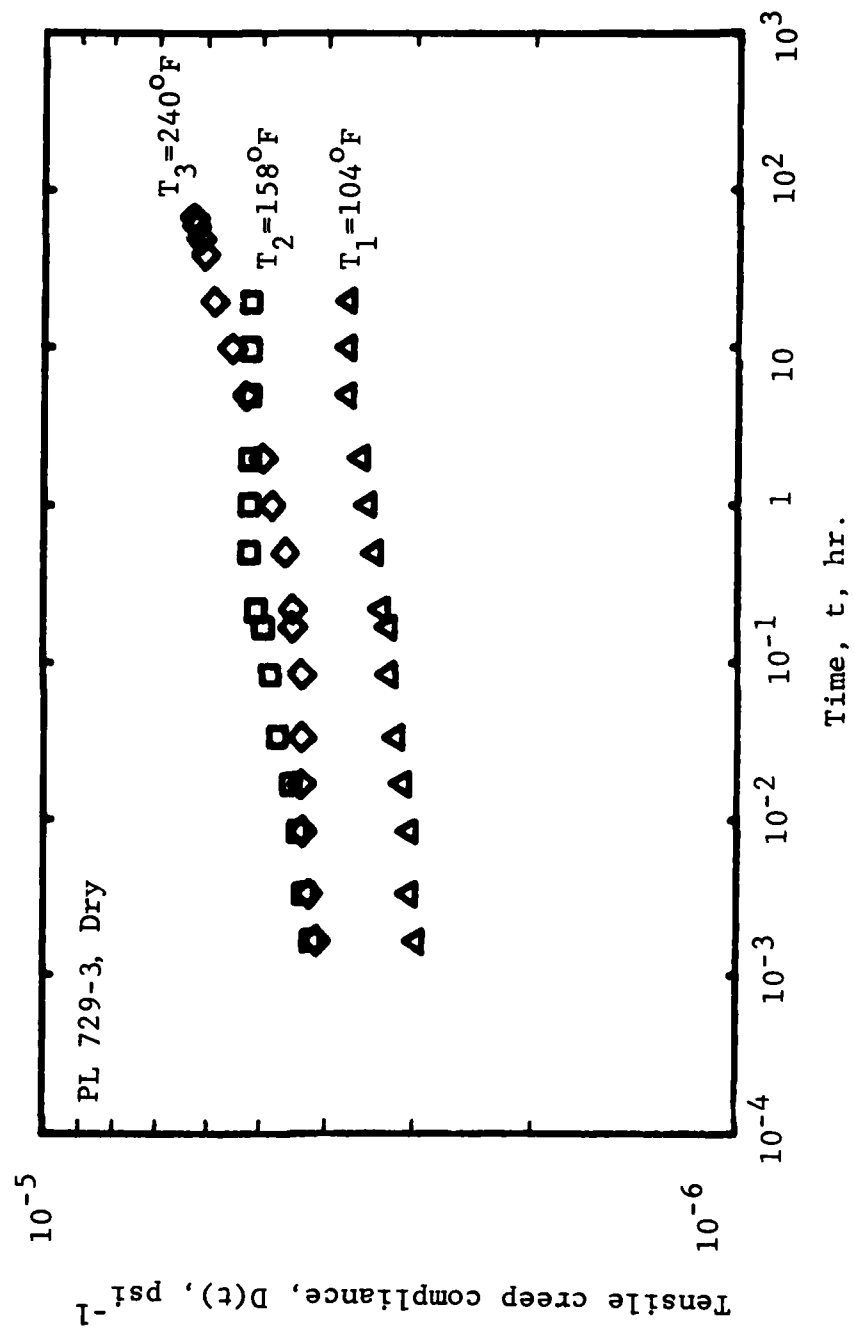


Figure A28 Tensile Creep Compliance of Dry PL 729-3

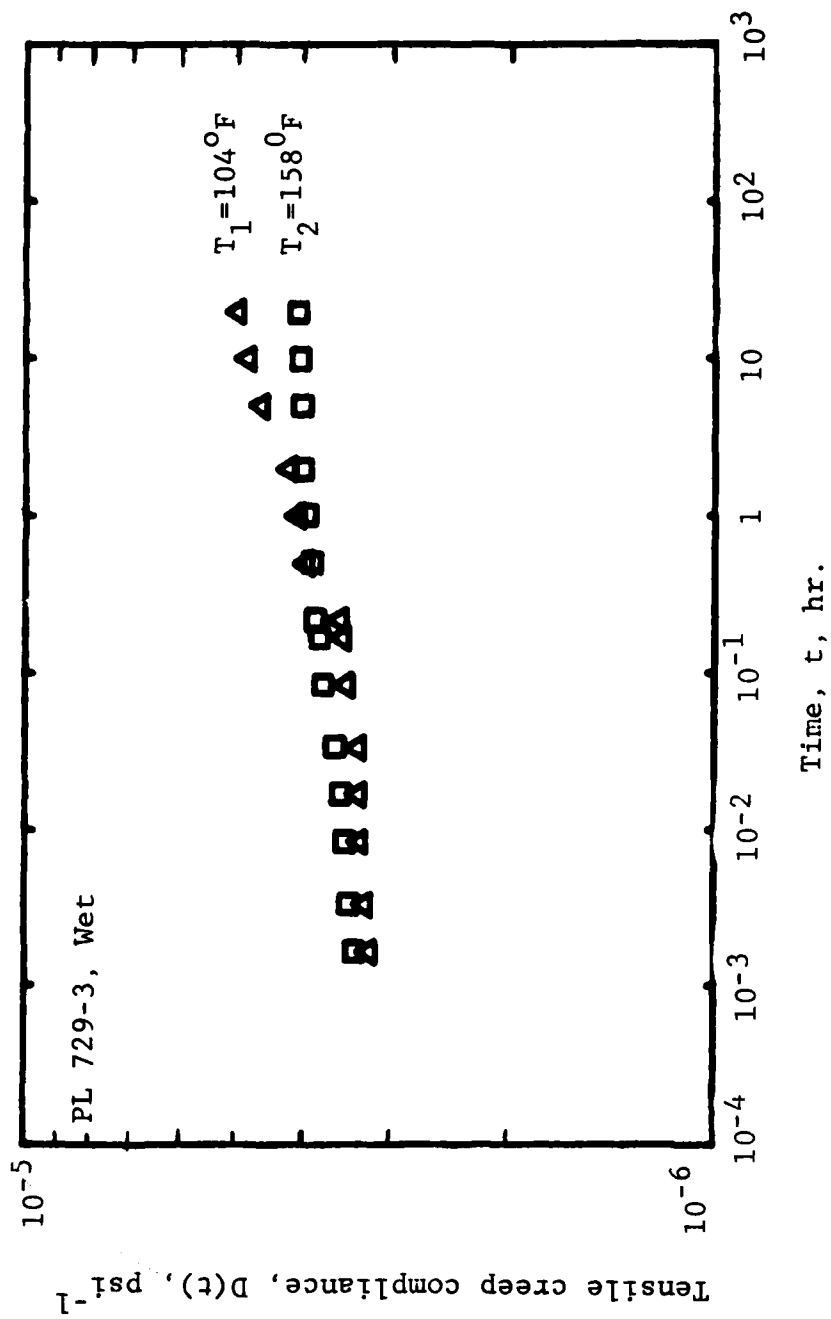


Figure A29 Tensile Creep Compliance of Wet PL 729-3



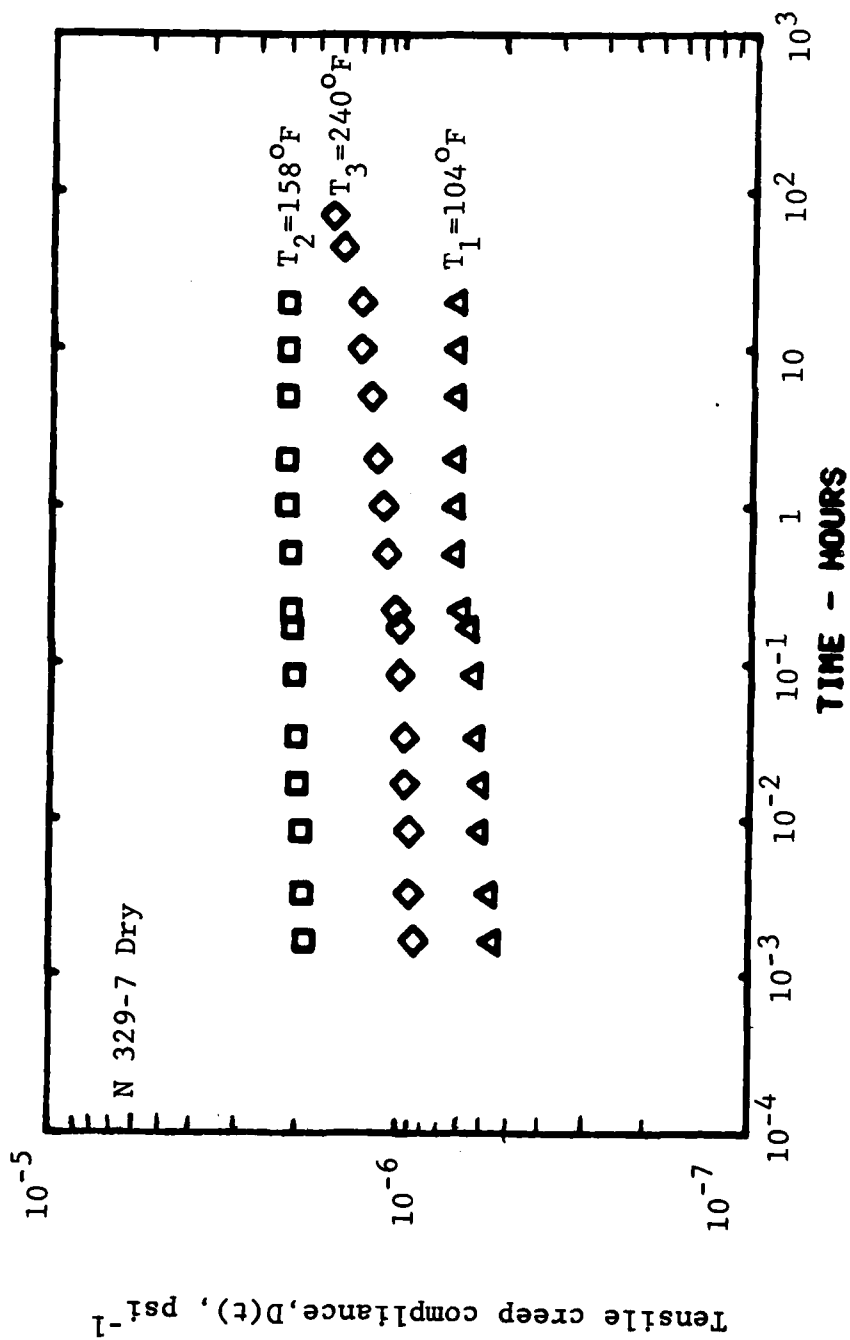


Figure A30 Tensile Creep Compliance of Dry N 329-7

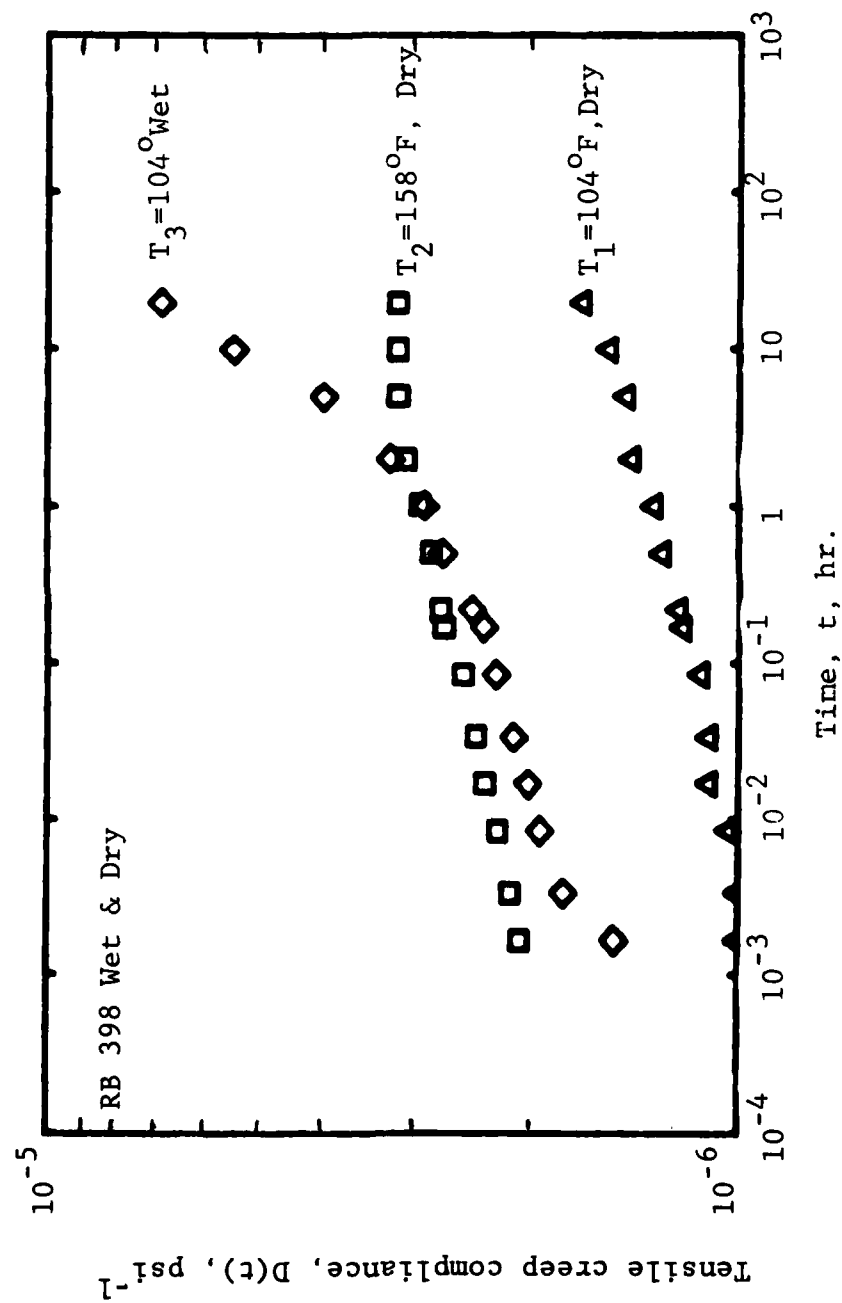


Figure A31 Tensile Creep Compliance of Dry and Wet RB 398

# FM-400 DRY 2000 PSI

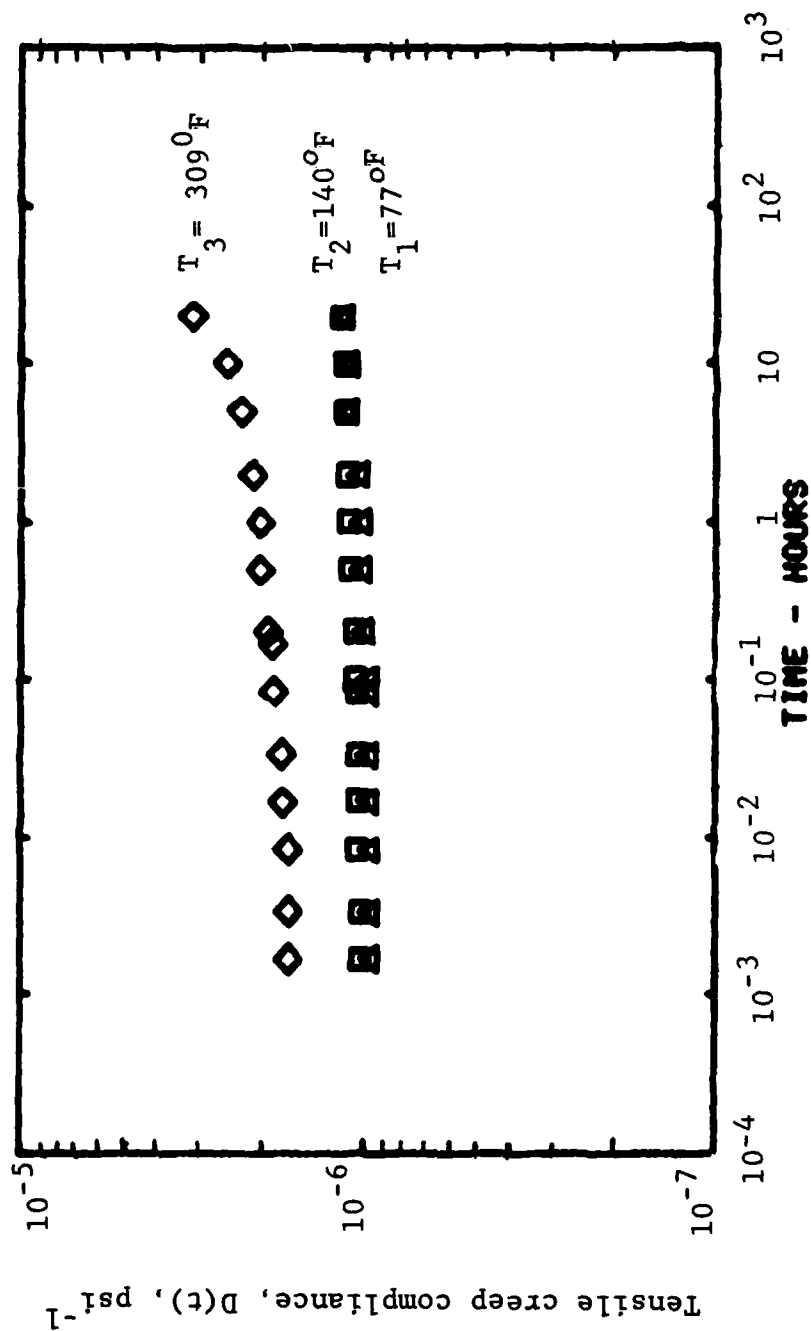


Figure A32 Tensile Creep Compliance of Dry FM-400

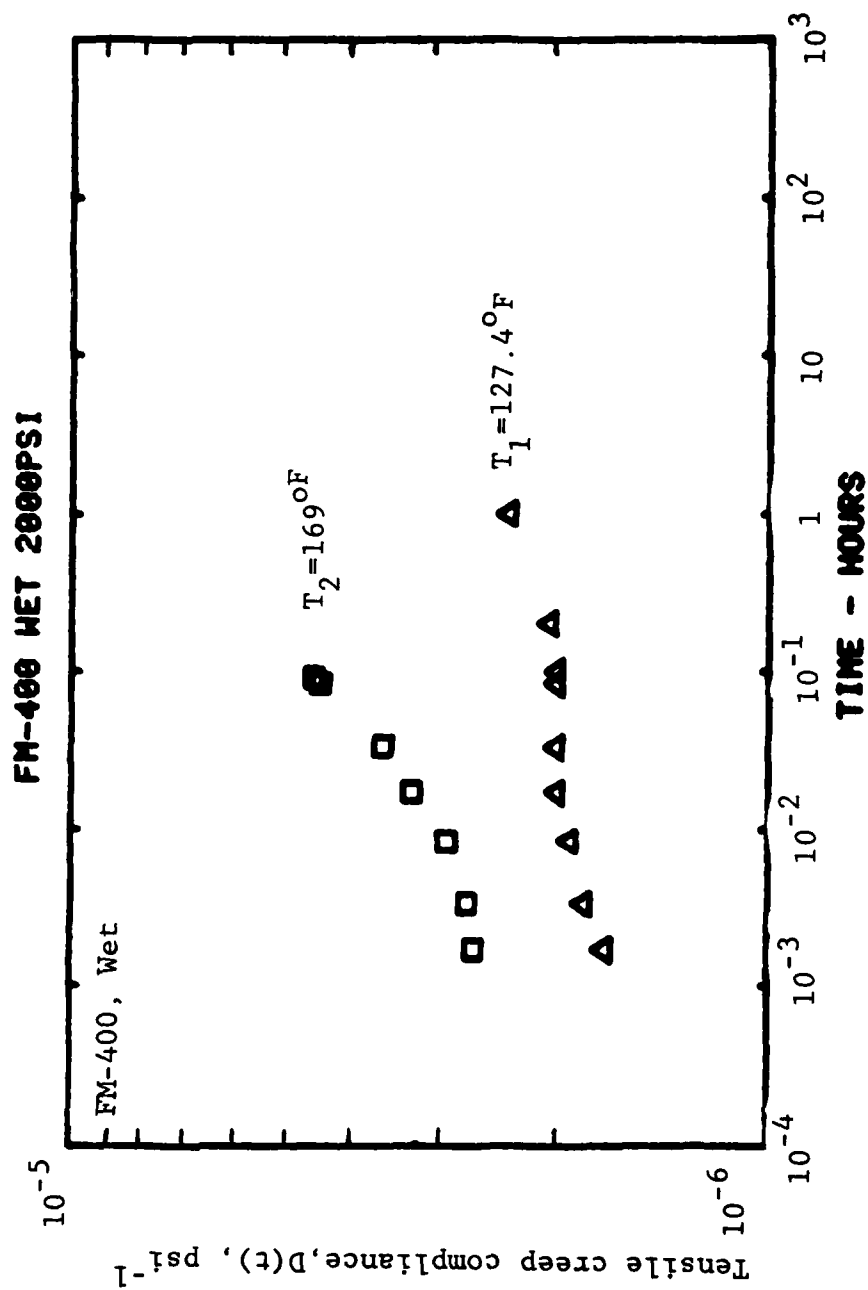


Figure A33 Tensile Creep Compliance of Wet FM-400

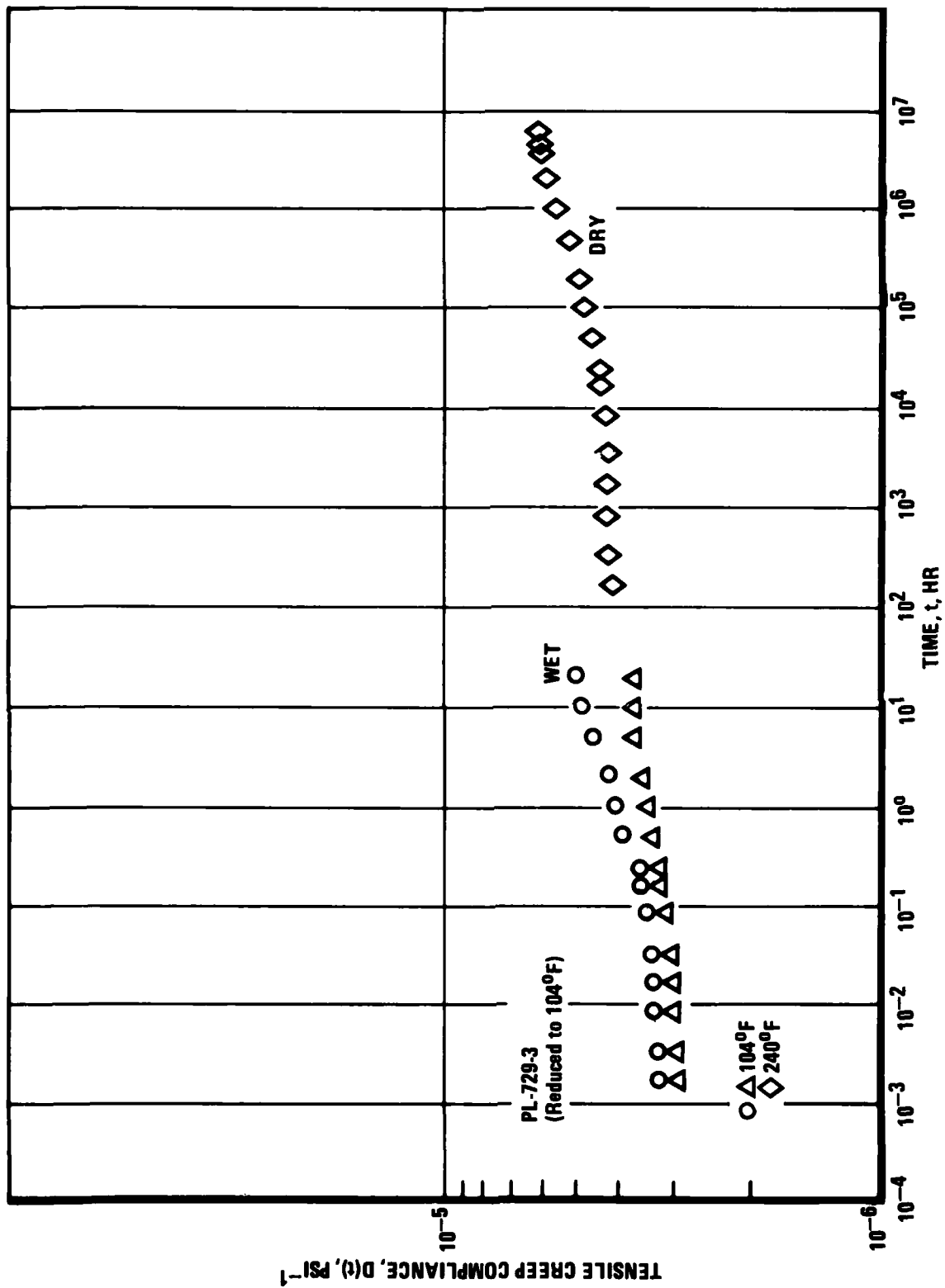


Figure A34 Tensile Creep Compliance of PL 729-3, Dry and Wet, (Reduced to 104°F)

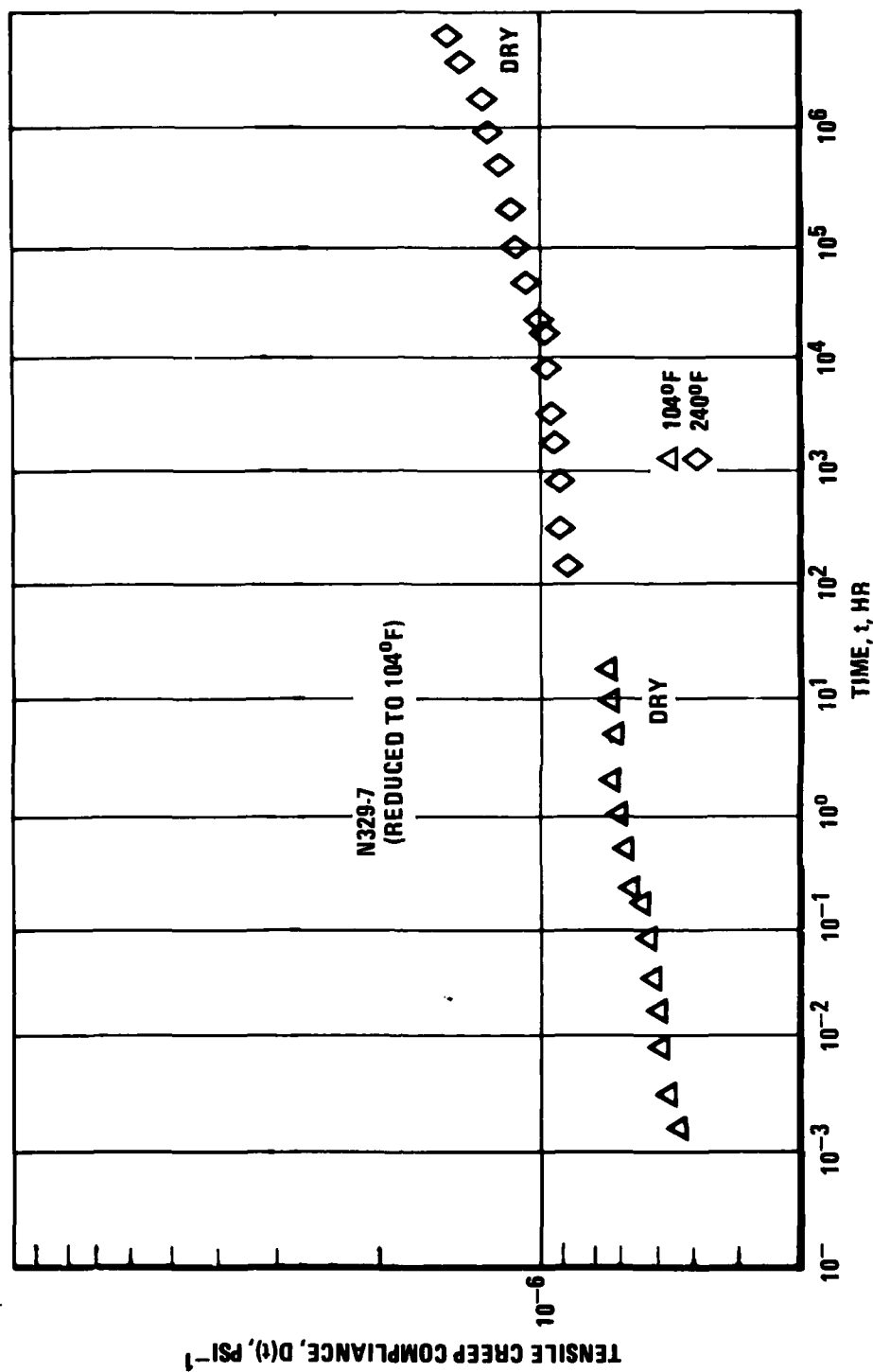


Figure A35 Tensile Creep Compliance of Dry N 329-7 (Reduced to  $104^{\circ}\text{F}$ )

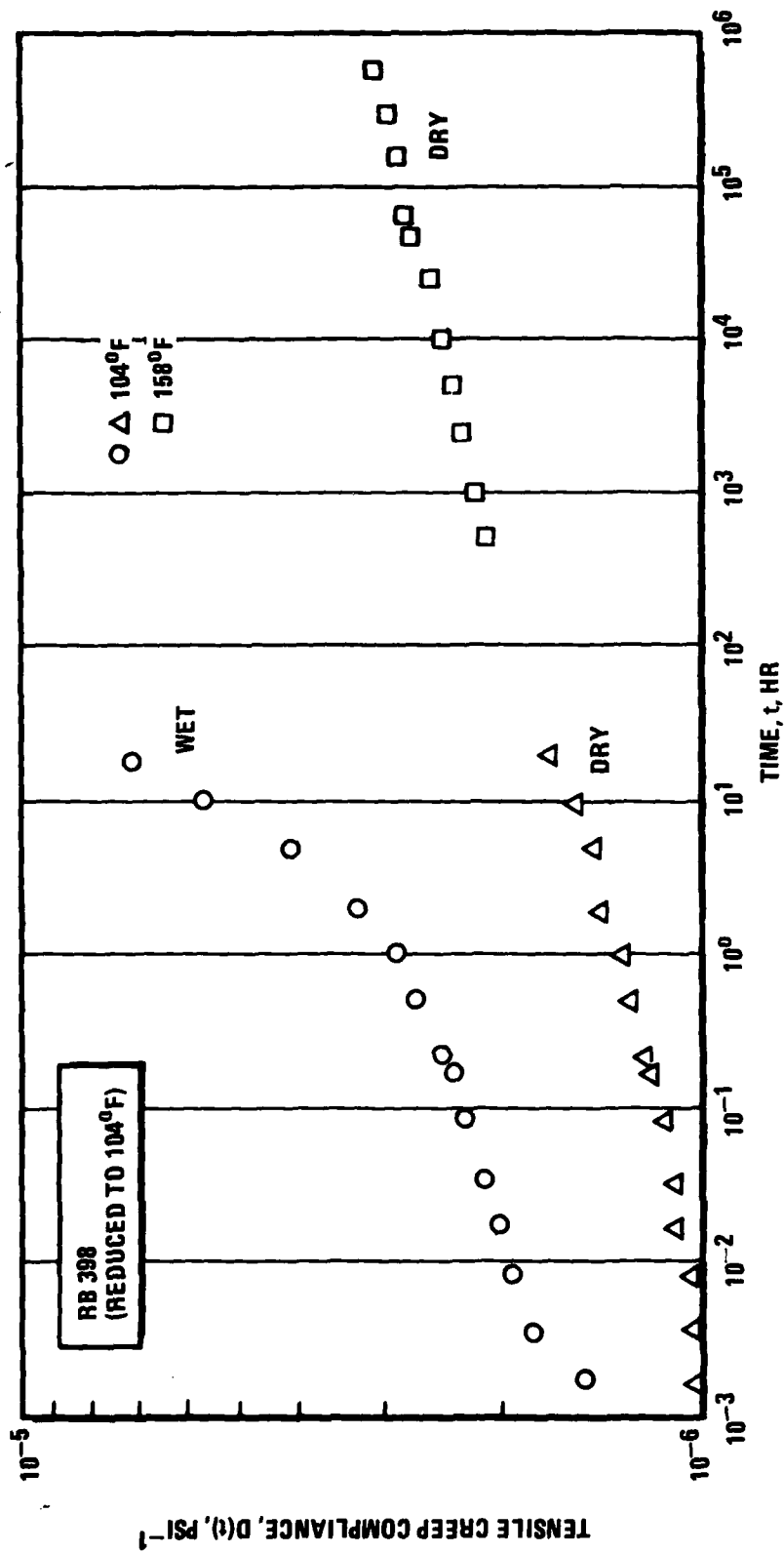


Figure A36 Tensile Creep Compliance of RB-398, Dry and Wet, (Reduced to 104°F)

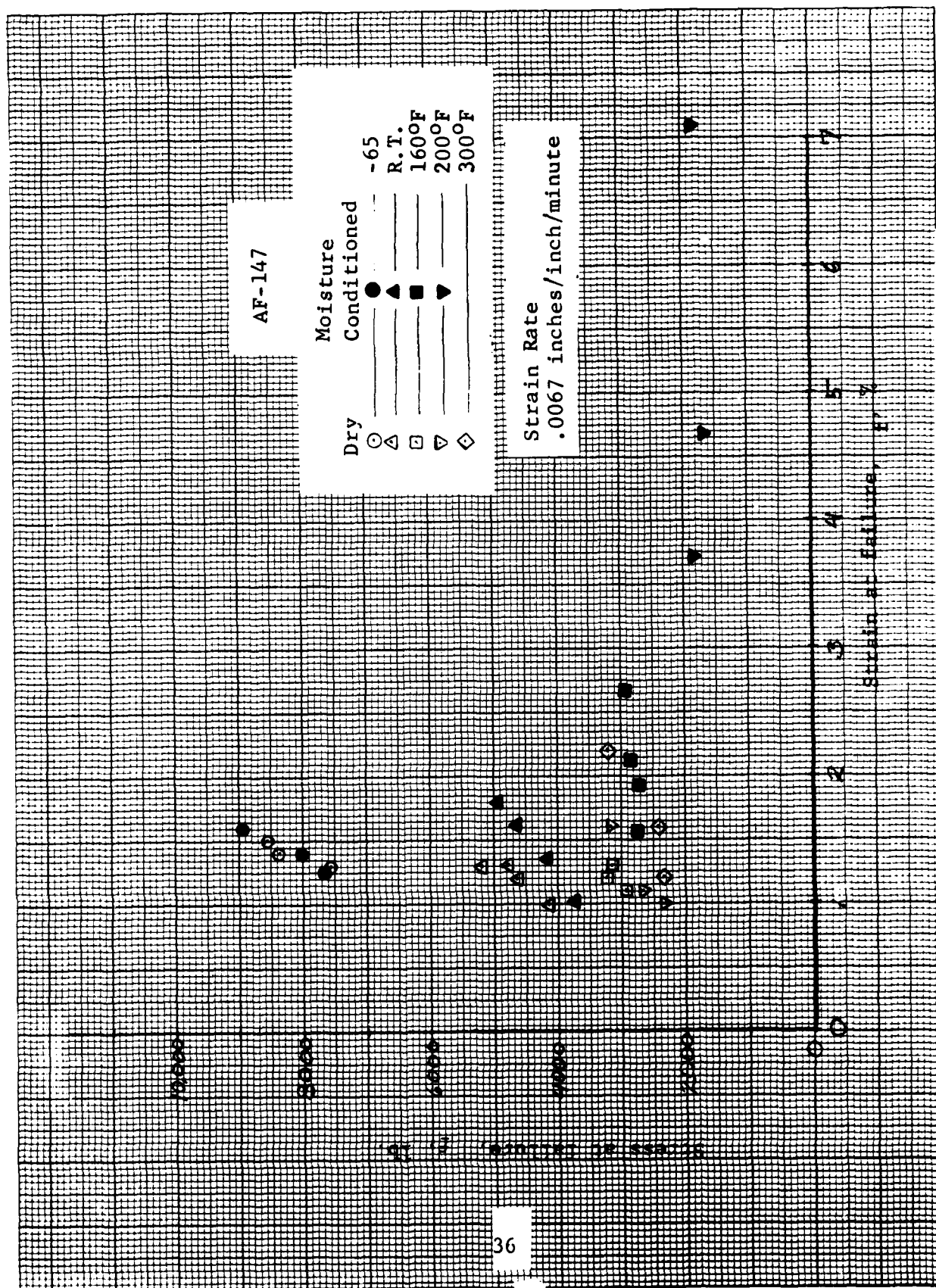


Figure A37 Smith Plots for AF-147 Adhesive



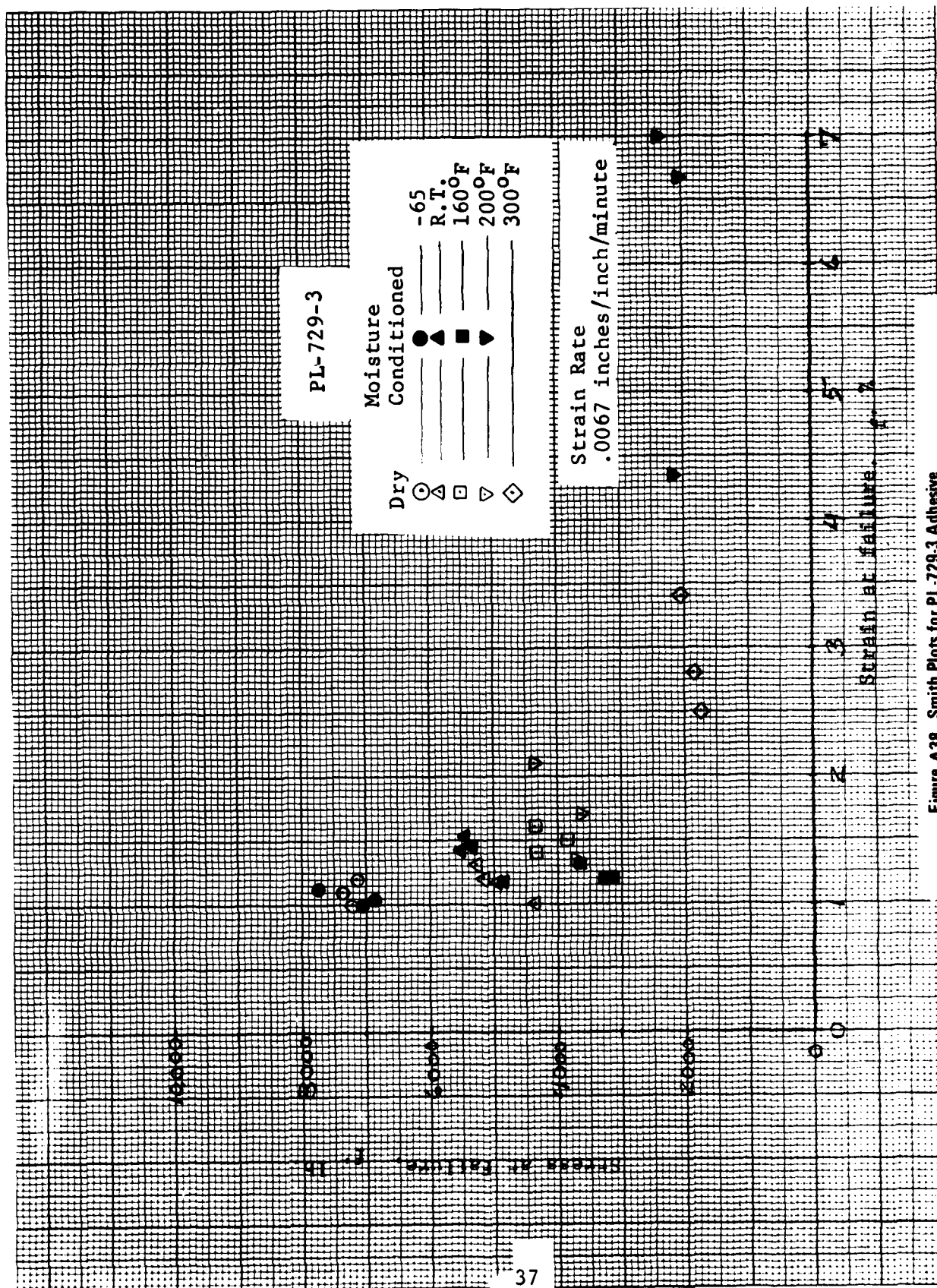


Figure A38 Smith Plots for PL-729-3 Adhesive

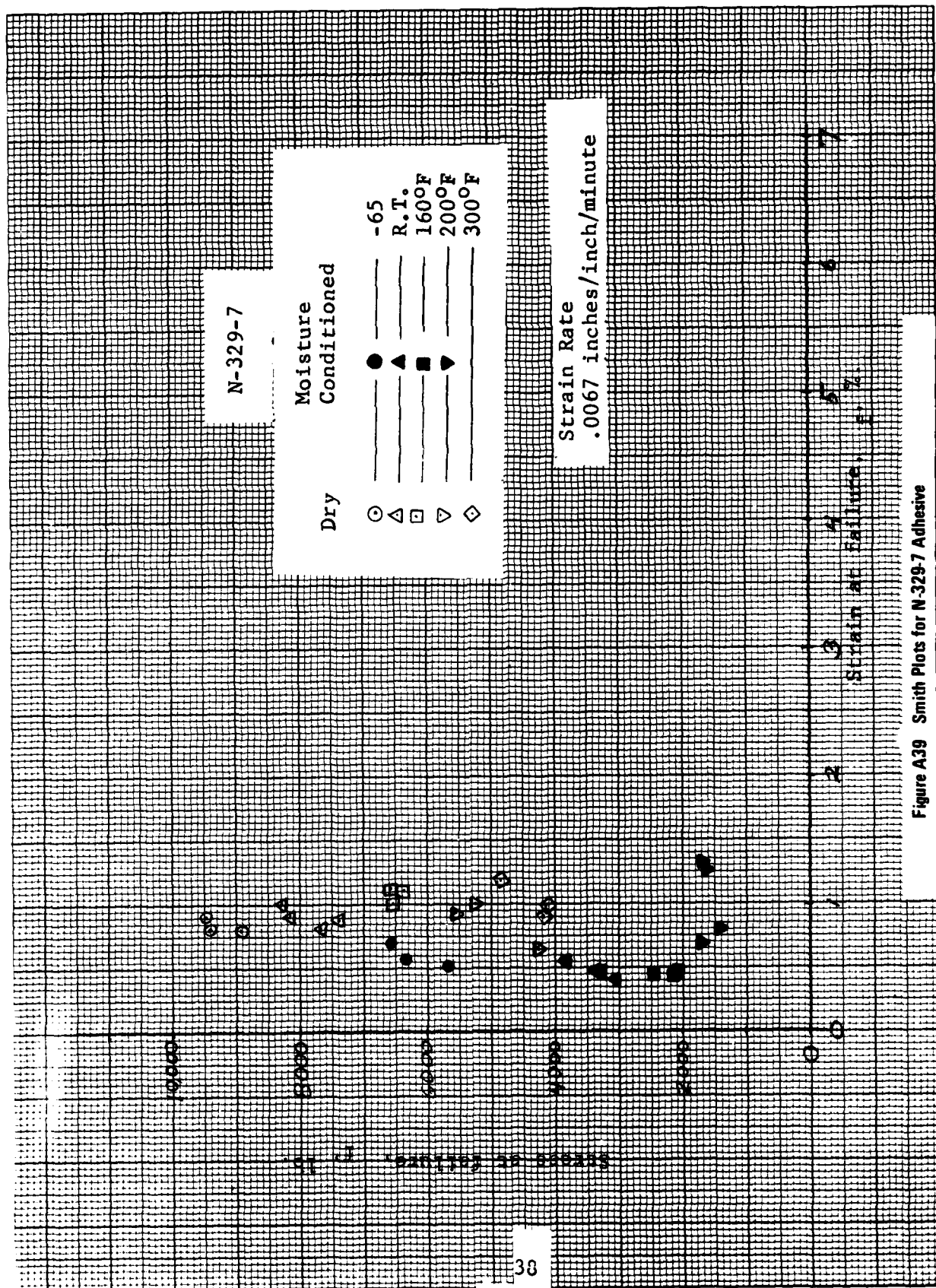


Figure A39 Smith Plots for N-329-7 Adhesive

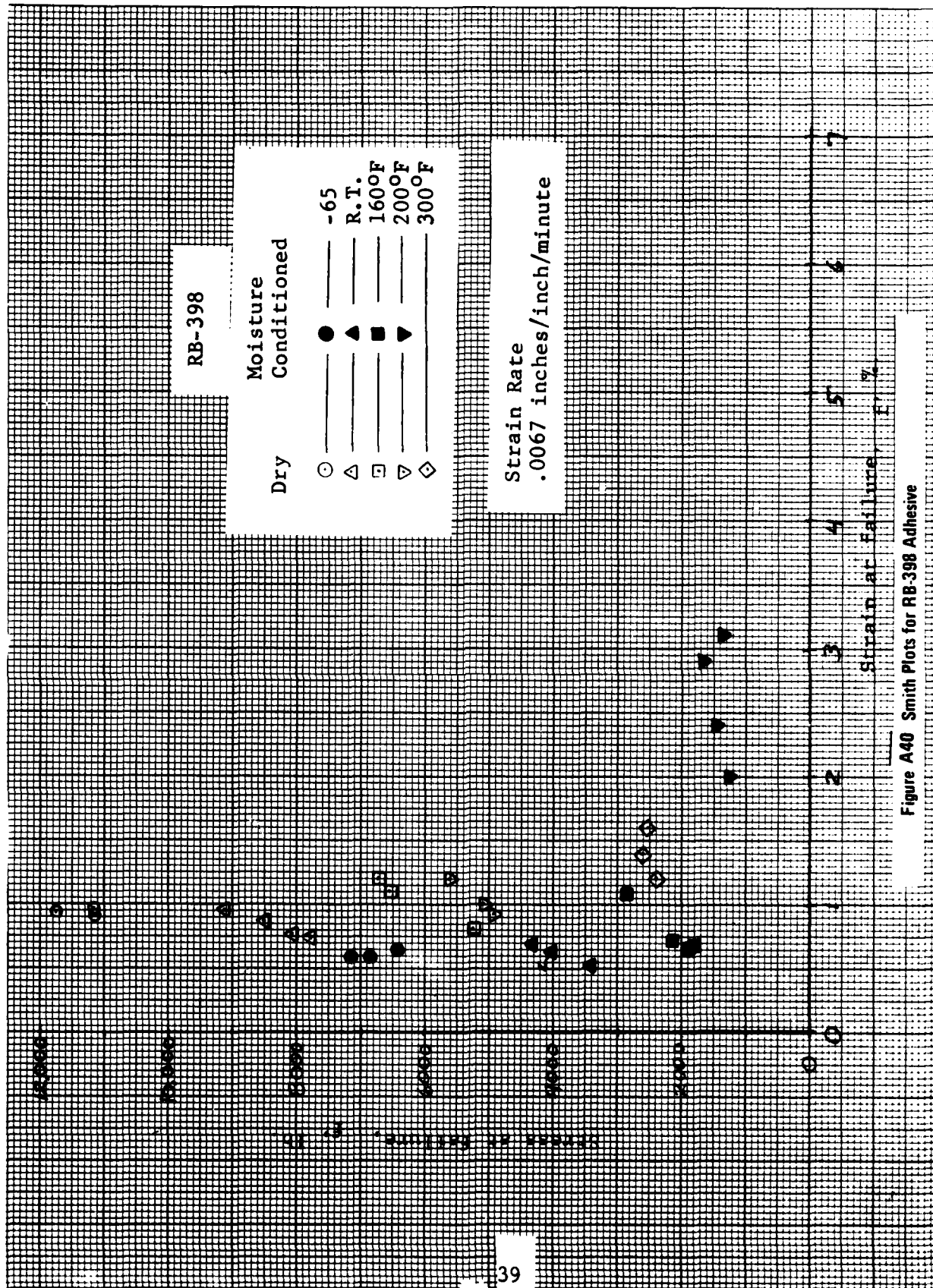
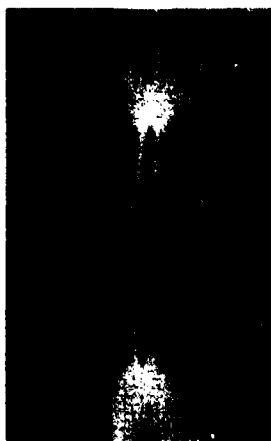


Figure A40 Smith Plots for RB-398 Adhesive

## 5.0



0.543"  
TO  
0.545"



0.513"  
TO  
0.520"



**P-6**

**N-1**

**SPECIMEN N-2**

**Figure A41 Crack Shape at Selected Crack Lengths of Fracture Mechanics FM-73M Neat Adhesive Specimens Tested Dry at 140°F**

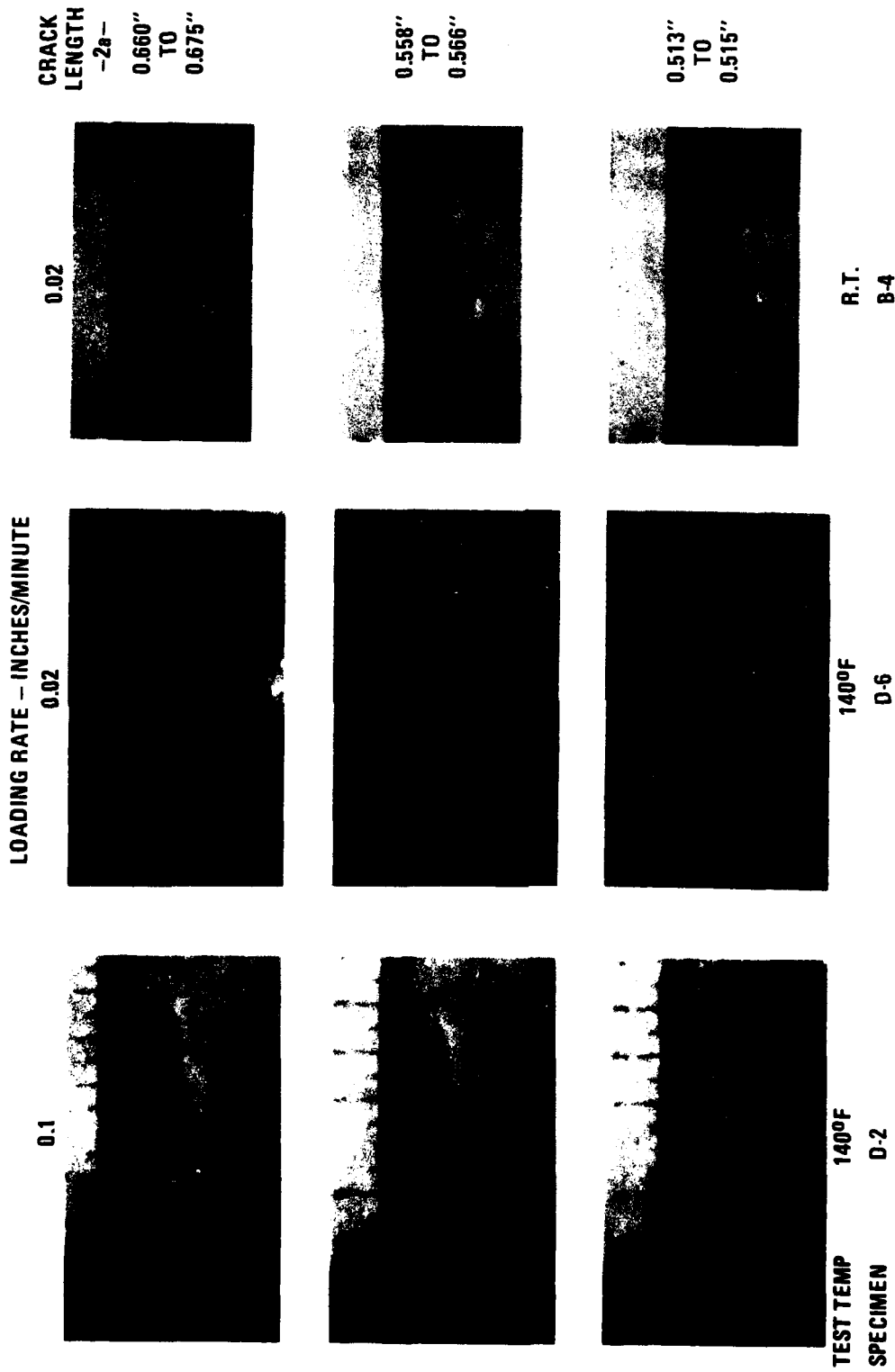


Figure A42 Crack Shapes at Selected Crack Lengths of Moisture Conditioned FM-73M Neat Adhesive Specimens Tested at Room Temperature and 140°F

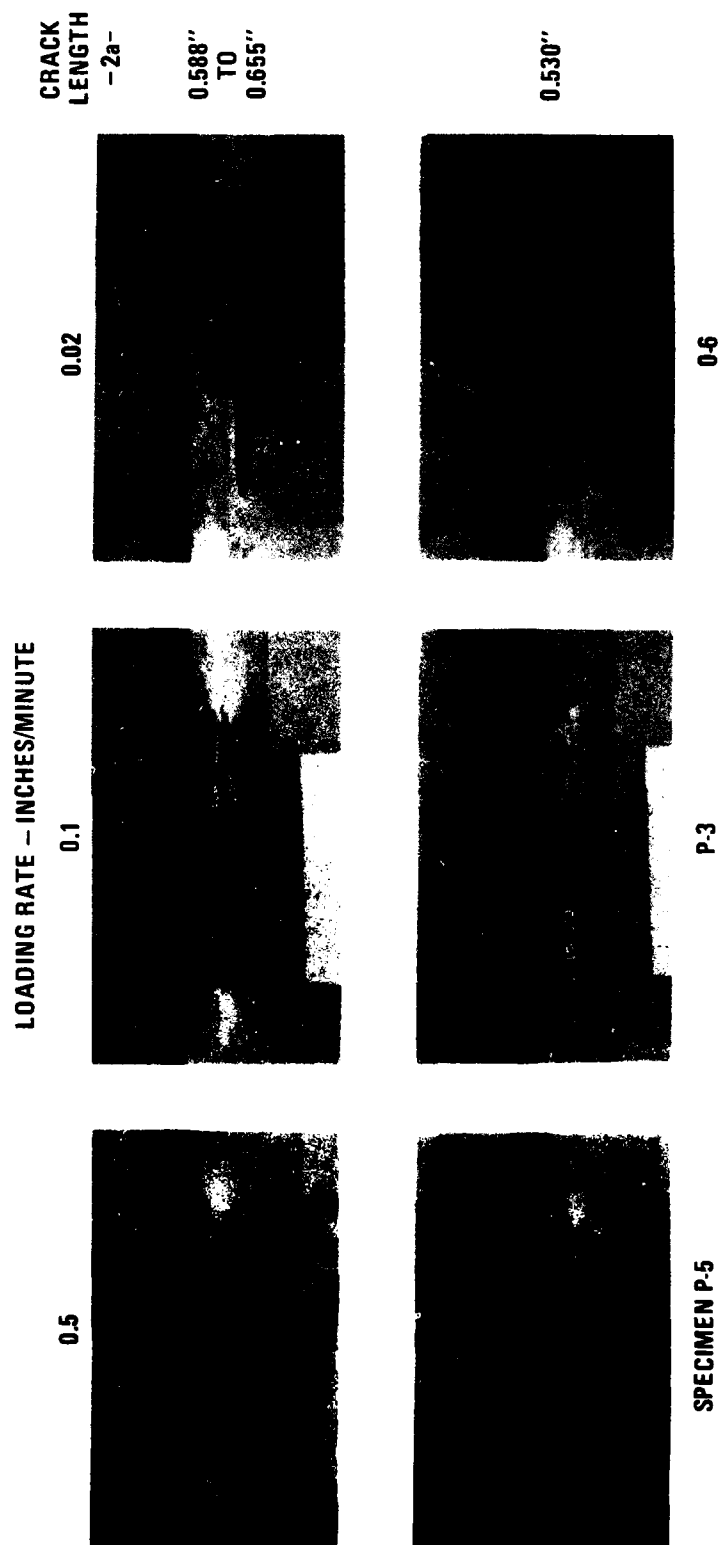


Figure A43 Crack Shape at Selected Crack Lengths of Fracture Mechanics FM-73M Neat Adhesives Specimens Tested Dry at Room Temperature

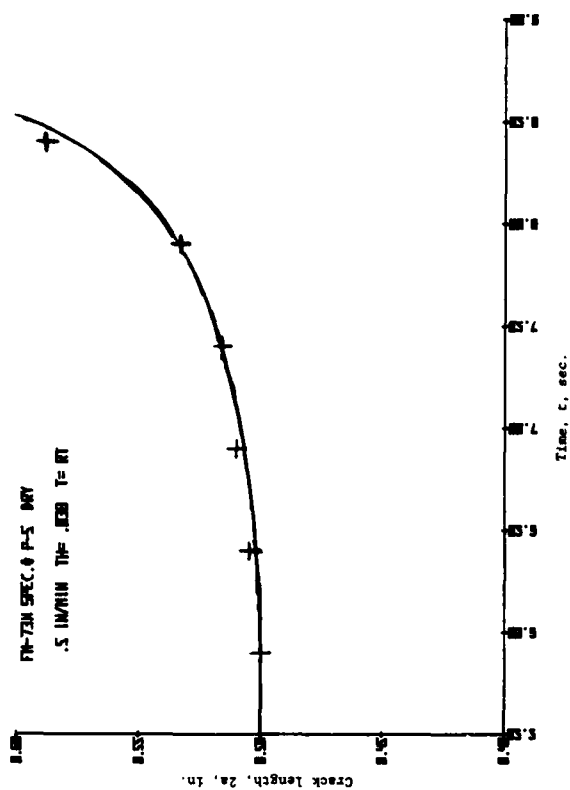


Figure A45 Crack Growth in Dry FM-73M at 750°F

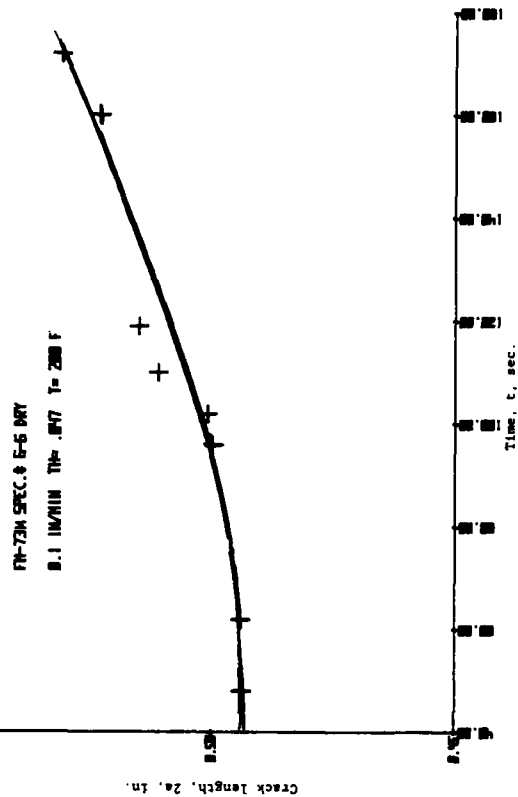


Figure A47 Crack Growth in Dry FM-73M at 200°F

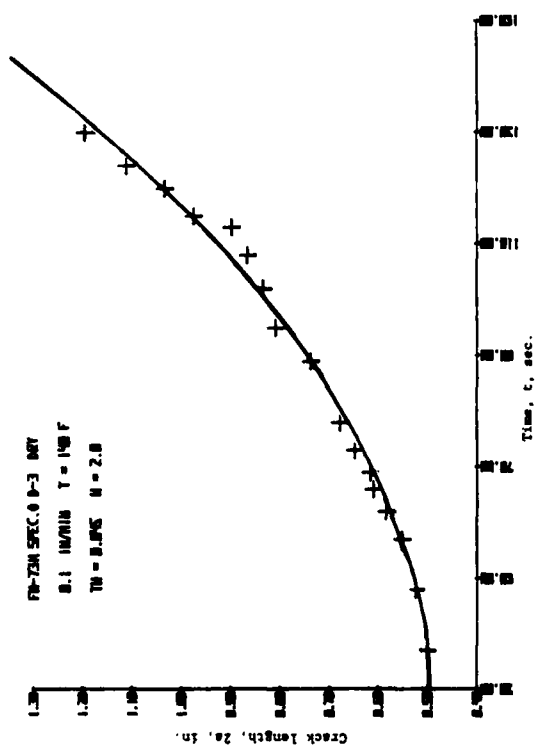


Figure A44 Crack Growth in Wet FM-73M at 140°F

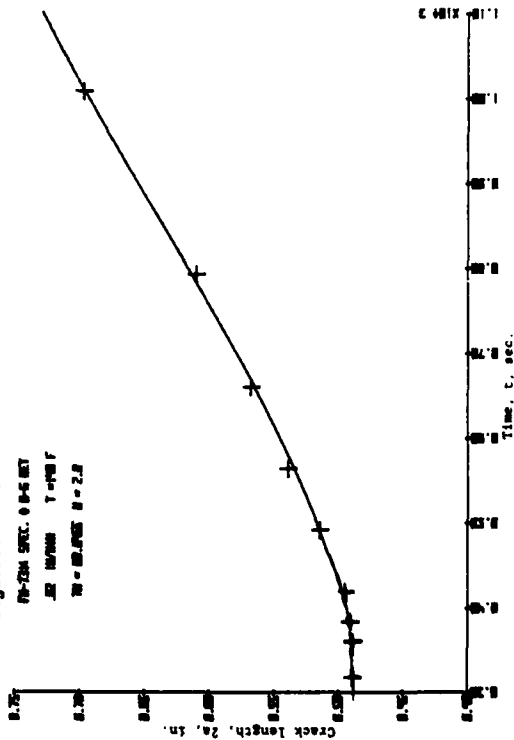


Figure A46 Crack Growth in Wet FM-73M at 140°F (2)

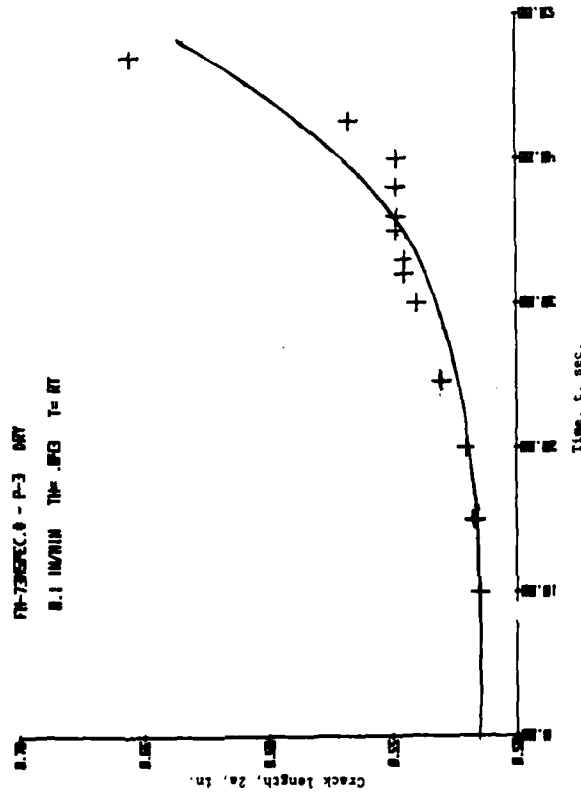
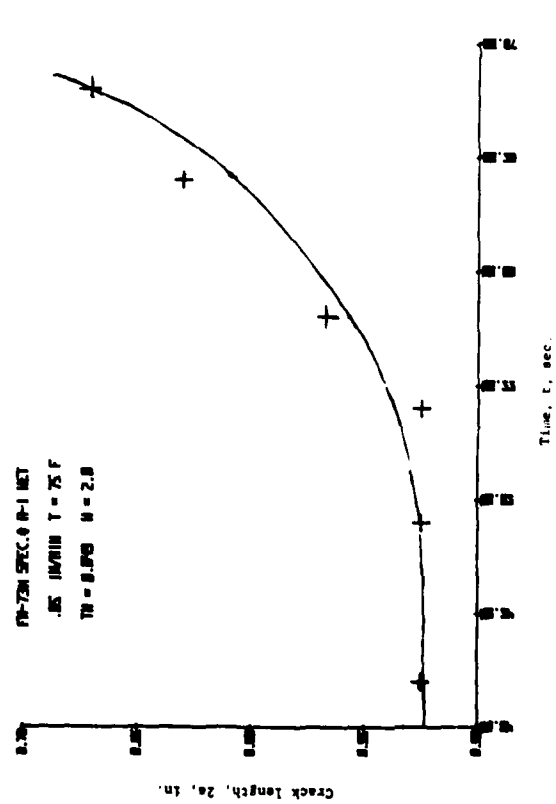


Figure A48 Crack Growth in Dry FM-73M at 750F (2)





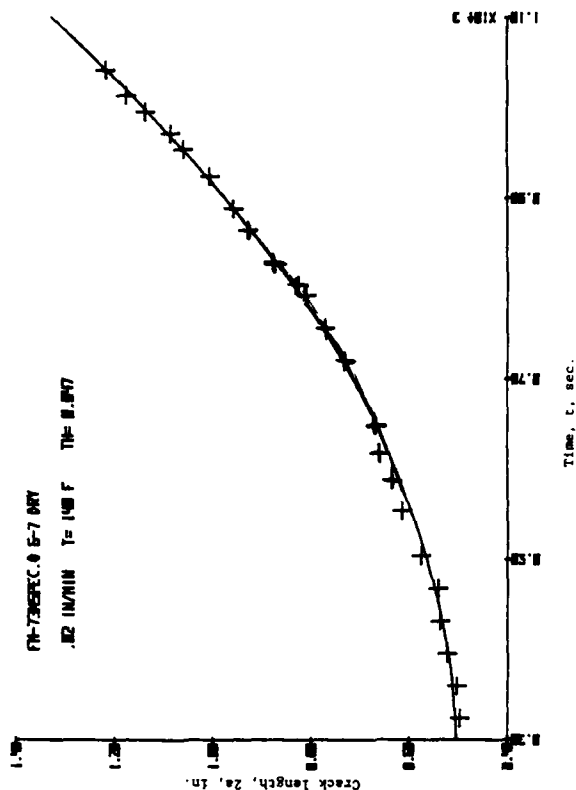


Figure A53 Crack Growth in Dry FM-73M at  $140^\circ F$

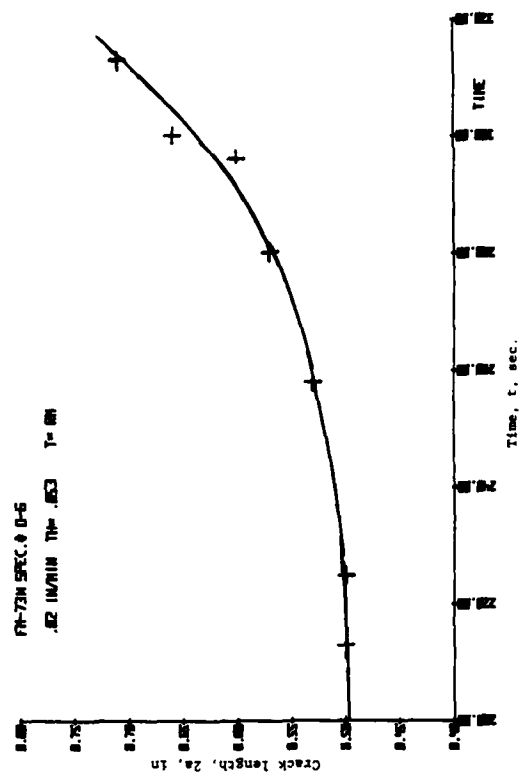


Figure A55 Crack Growth in Dry FM-73M at  $75^\circ F$  (5)

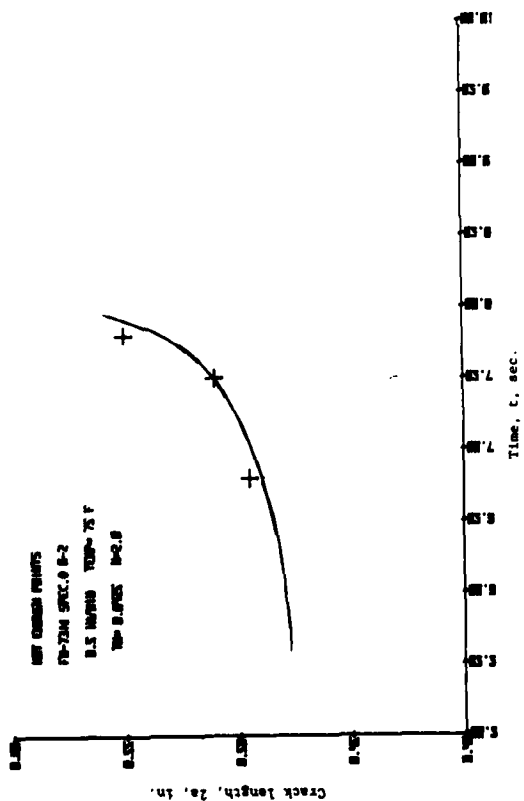


Figure A52 Crack Growth in Dry FM-73M at  $75^\circ F$  (4)

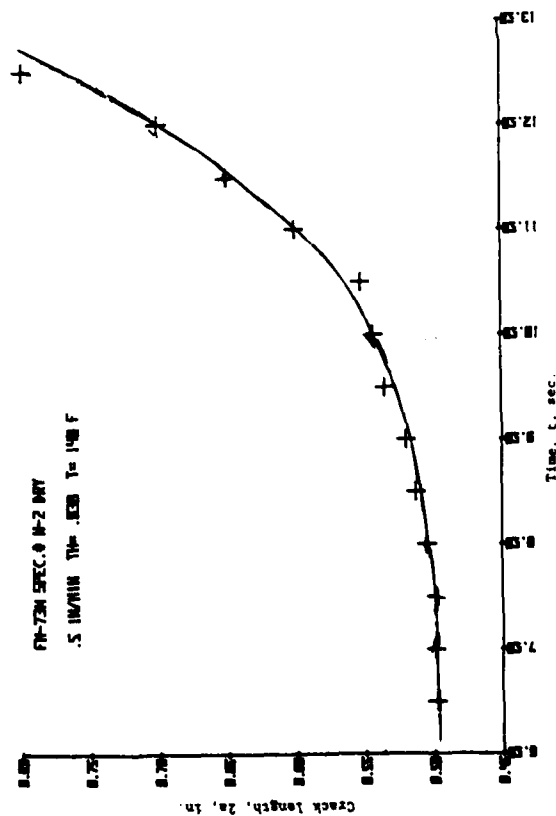


Figure A54 Crack Growth in Dry FM-73M at  $140^\circ F$  (2)

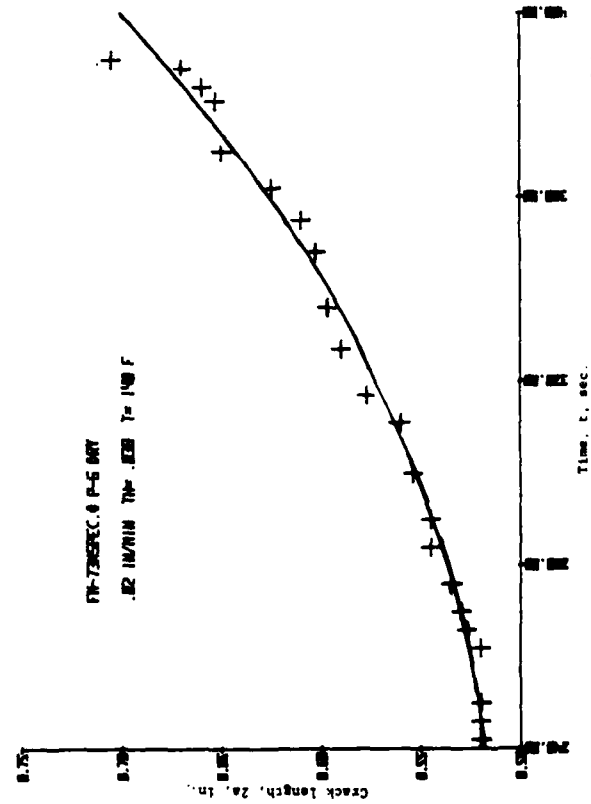


Figure A56 Crack Growth in Dry FM-73M at 140°F (2)

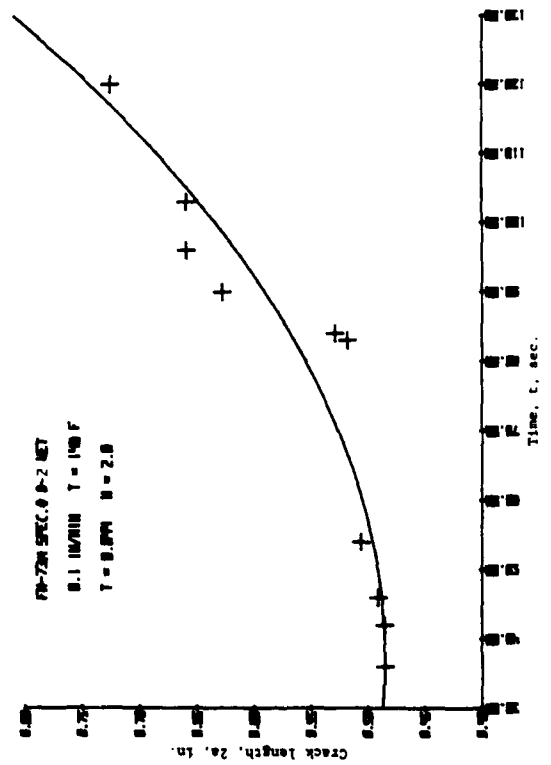


Figure A58 Crack Growth in Wet FM-73M at 140°F (4)

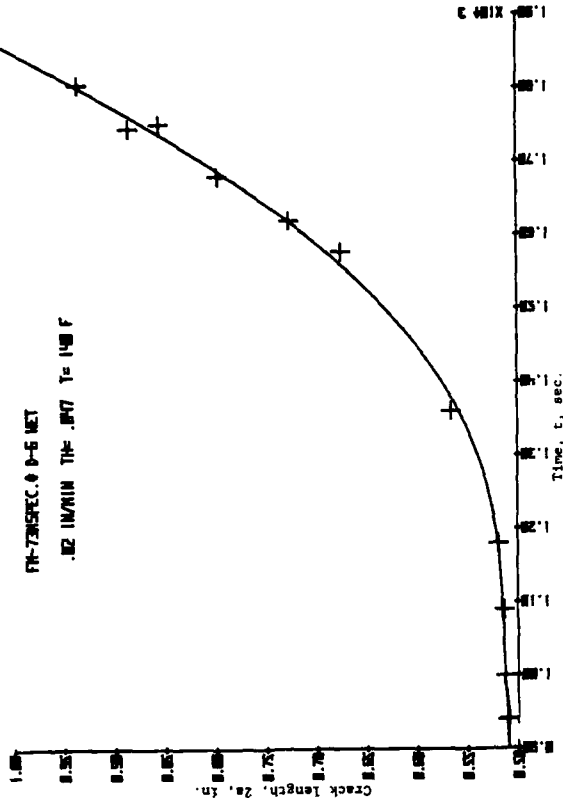


Figure A57 Crack Growth in Wet FM-73M at 140°F (3)

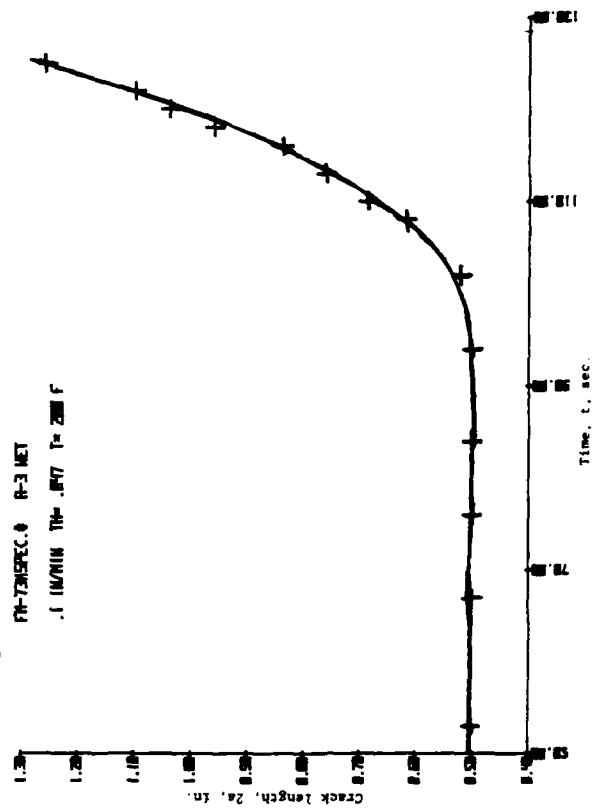


Figure A59 Crack Growth in Wet FM-73M at 200°F

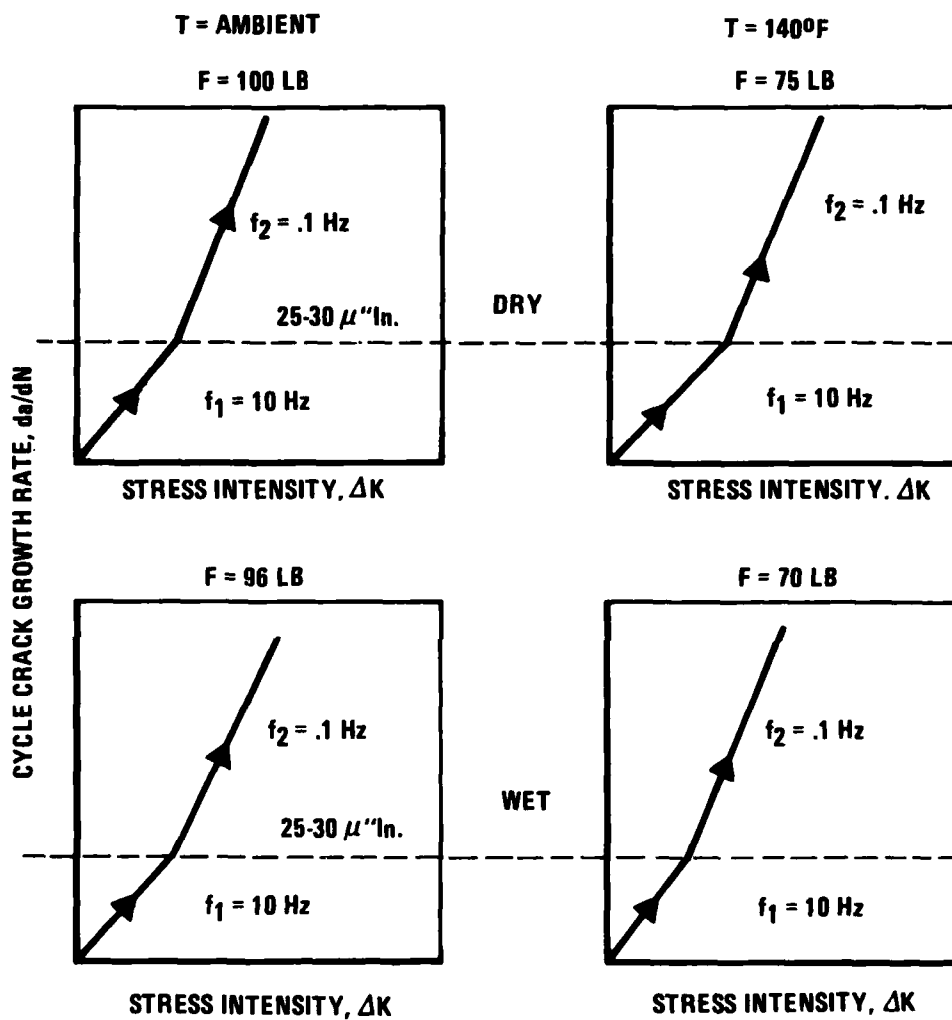
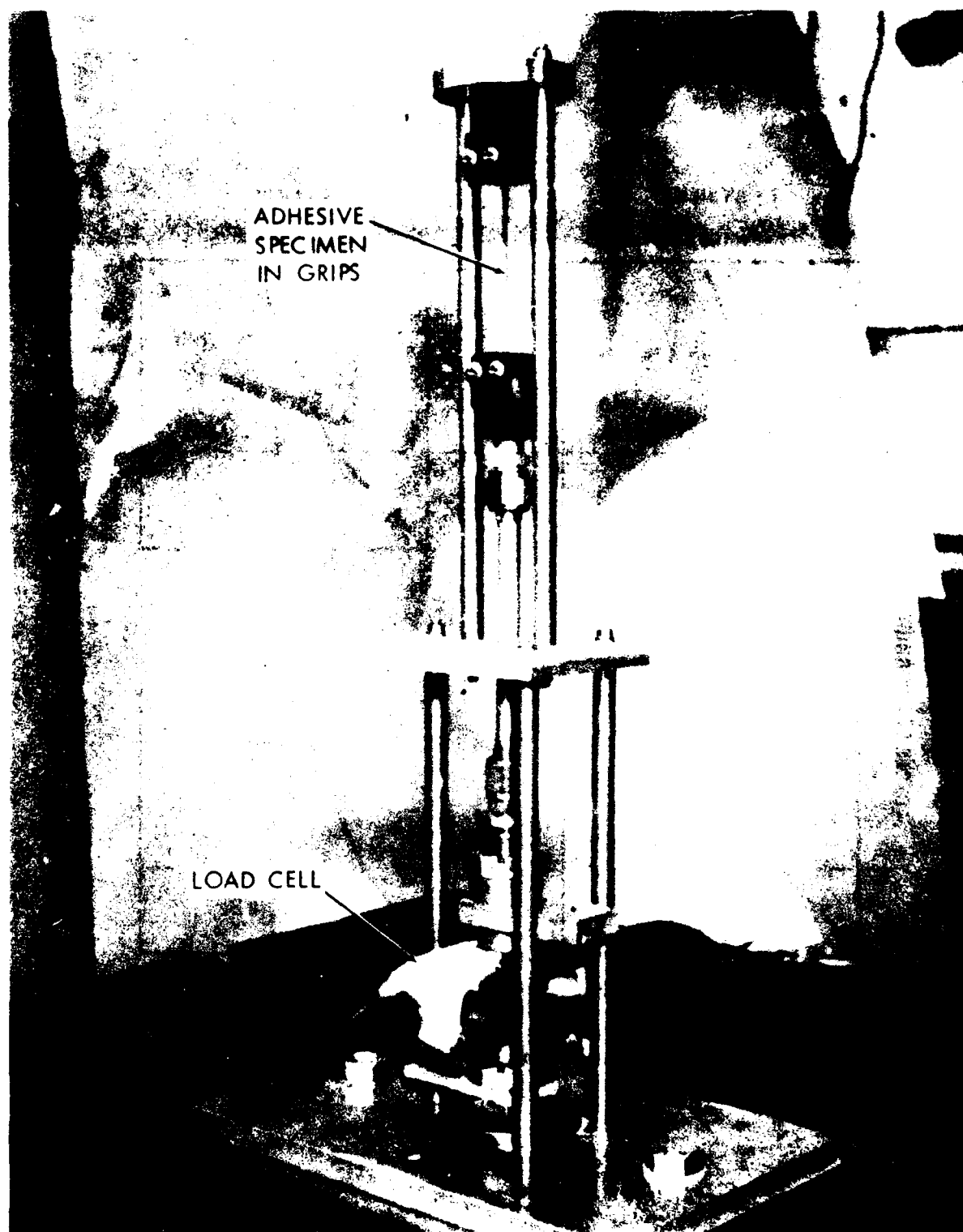


Figure A60 Cyclic Crack Growth Rate vs Stress Intensity Test Plan



**Figure A61 Loading Fixture for Fatigue Testing of Environmentally Conditioned Adhesive Materials**

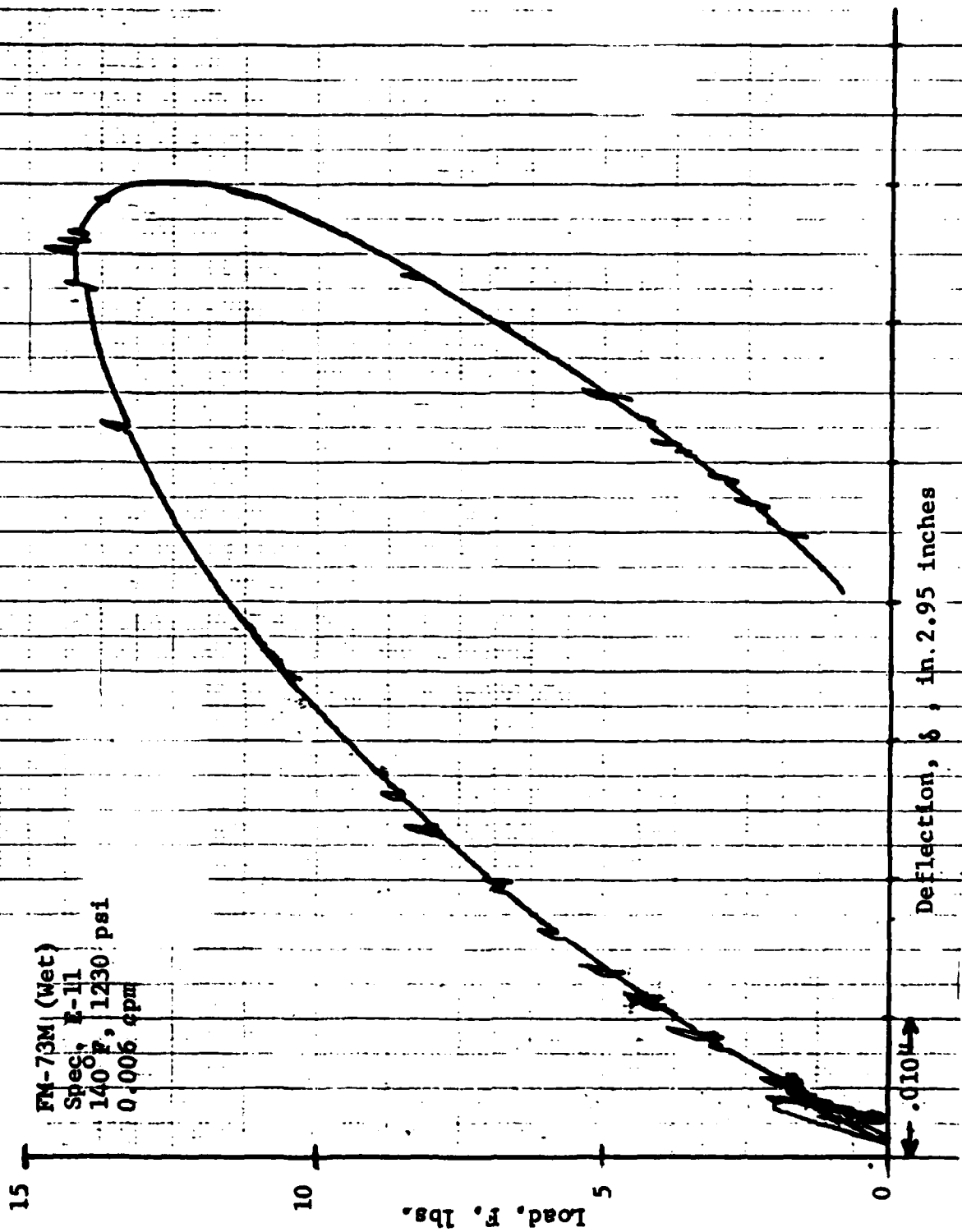


Figure A62 Load-Deflection History of Wet FM-73M Coupon, First Cycle of Loading and Unloading;  
Low Frequency

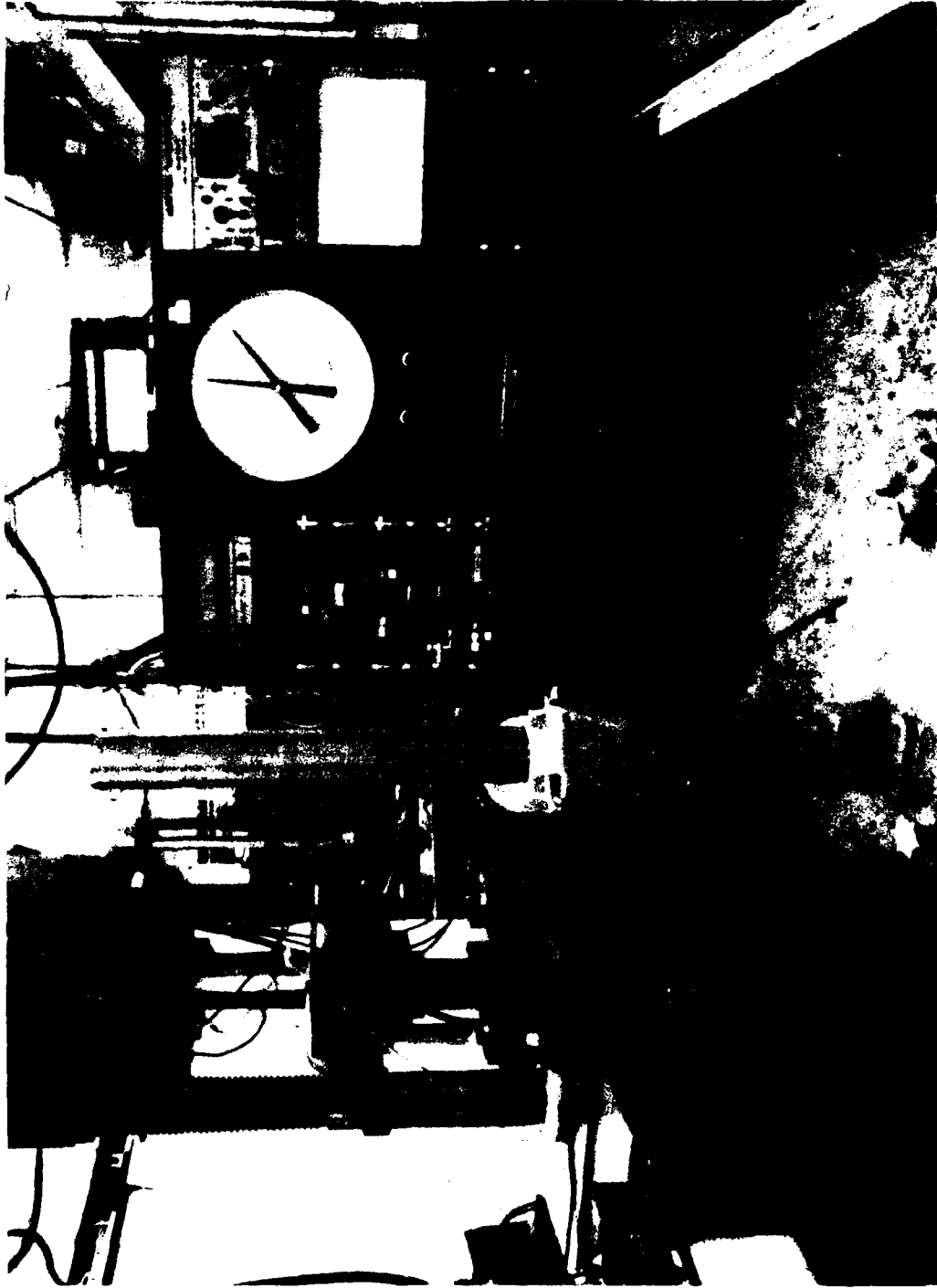


Figure A63 Fatigue Testing Equipment for Model Lap Shear Joints



Figure A64 FM-73M Adhesive Interlayer of Model Joint 12C11 After Adherend Etching, 10K Cycles

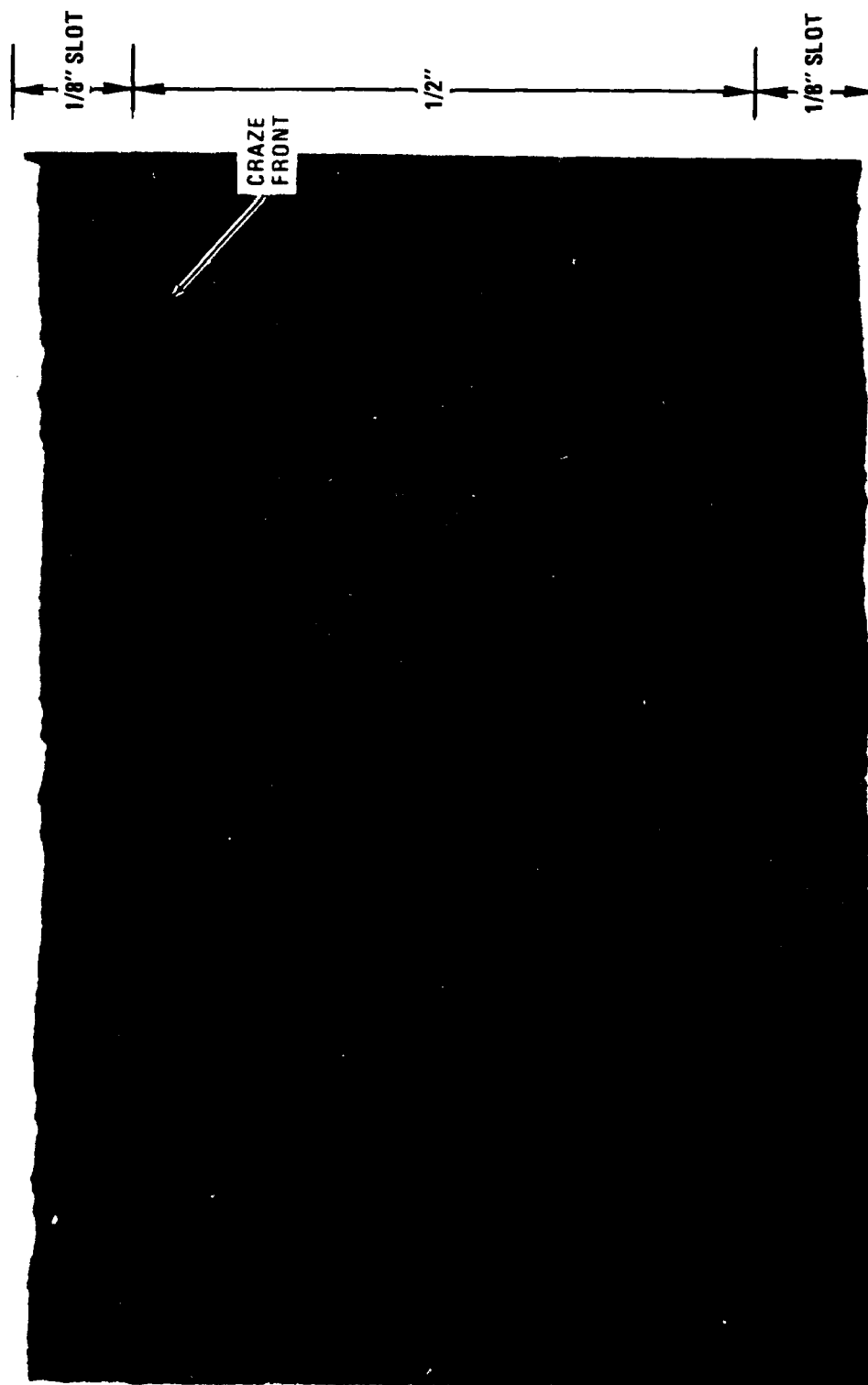


Figure A65 FM-73M Adhesive Interlayer of Model Joint 10C12 After Adherend Etching, 15K Cycles





Figure A66 FM-73M Adhesive Interlayer of Model Joint 12C12 After Adherend Etching, 20K Cycles



Figure A67 FM-73M Adhesive Interlayer of Failed Model Joint 3C10 After Adherend Etching, 27K Cycles

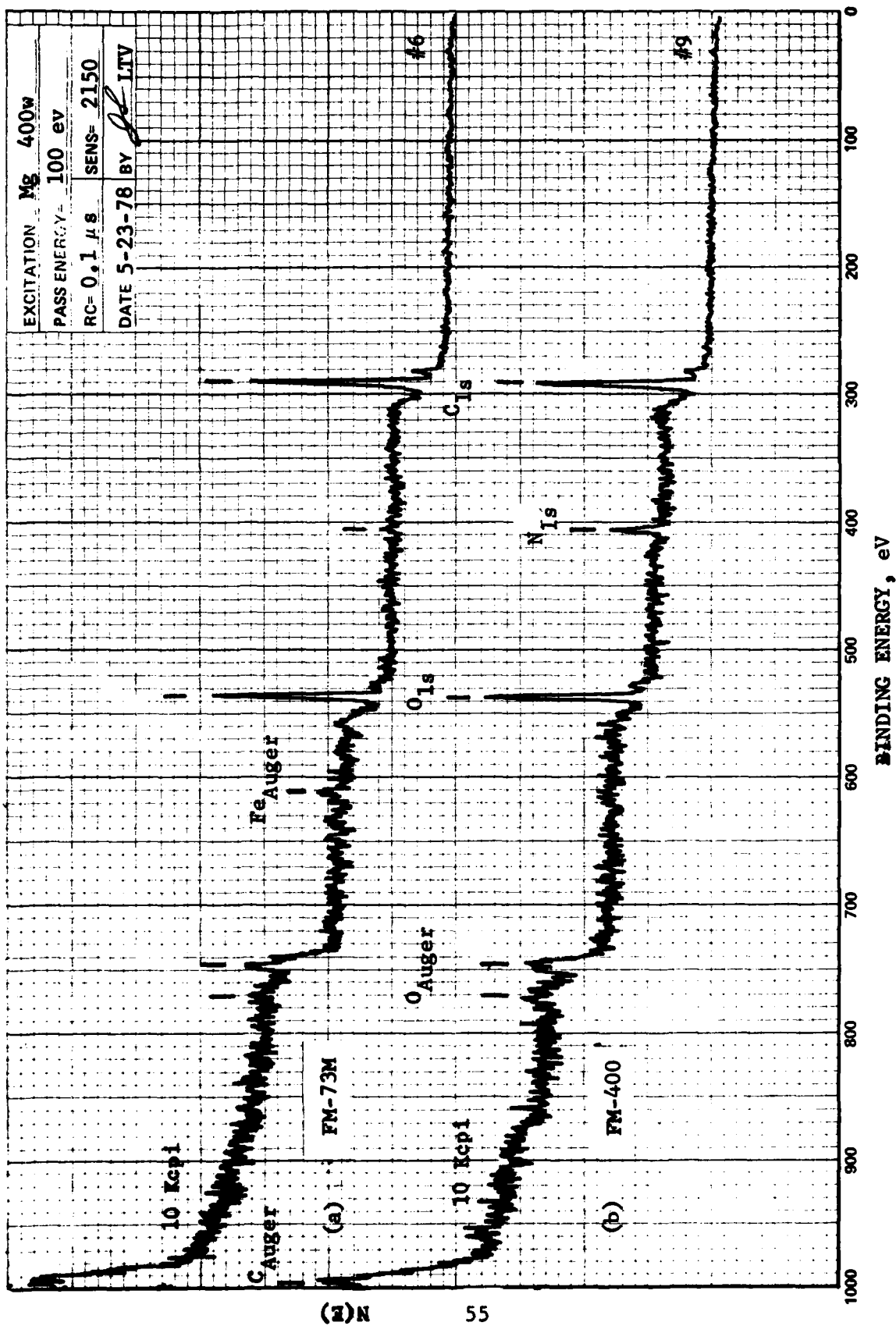


Figure A68 XPS Spectra of Dry FM-73M and FM-400 Model Joints



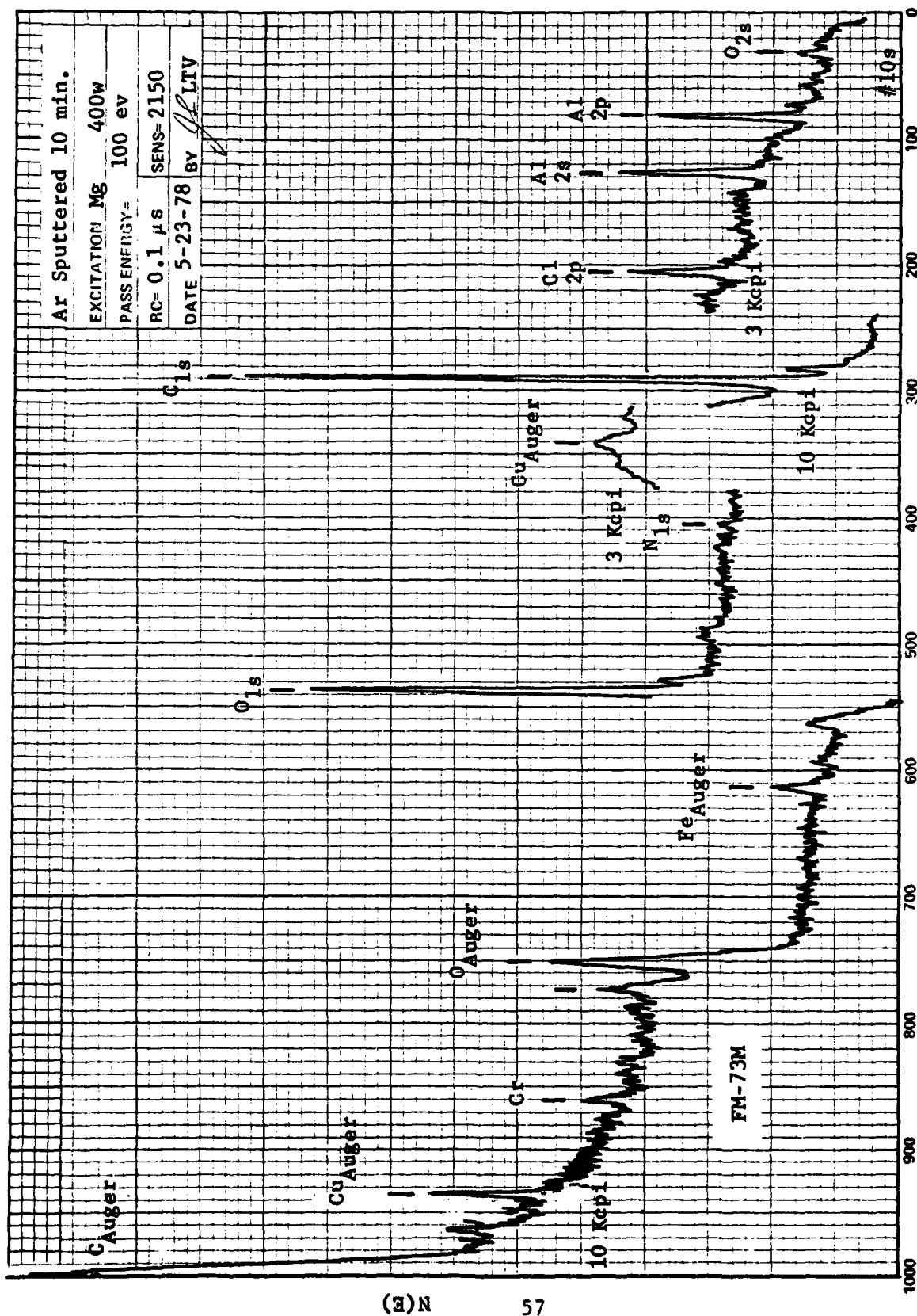


Figure A70 XPS Spectrum of Sputtered FM-73M Adhesive Interlayer

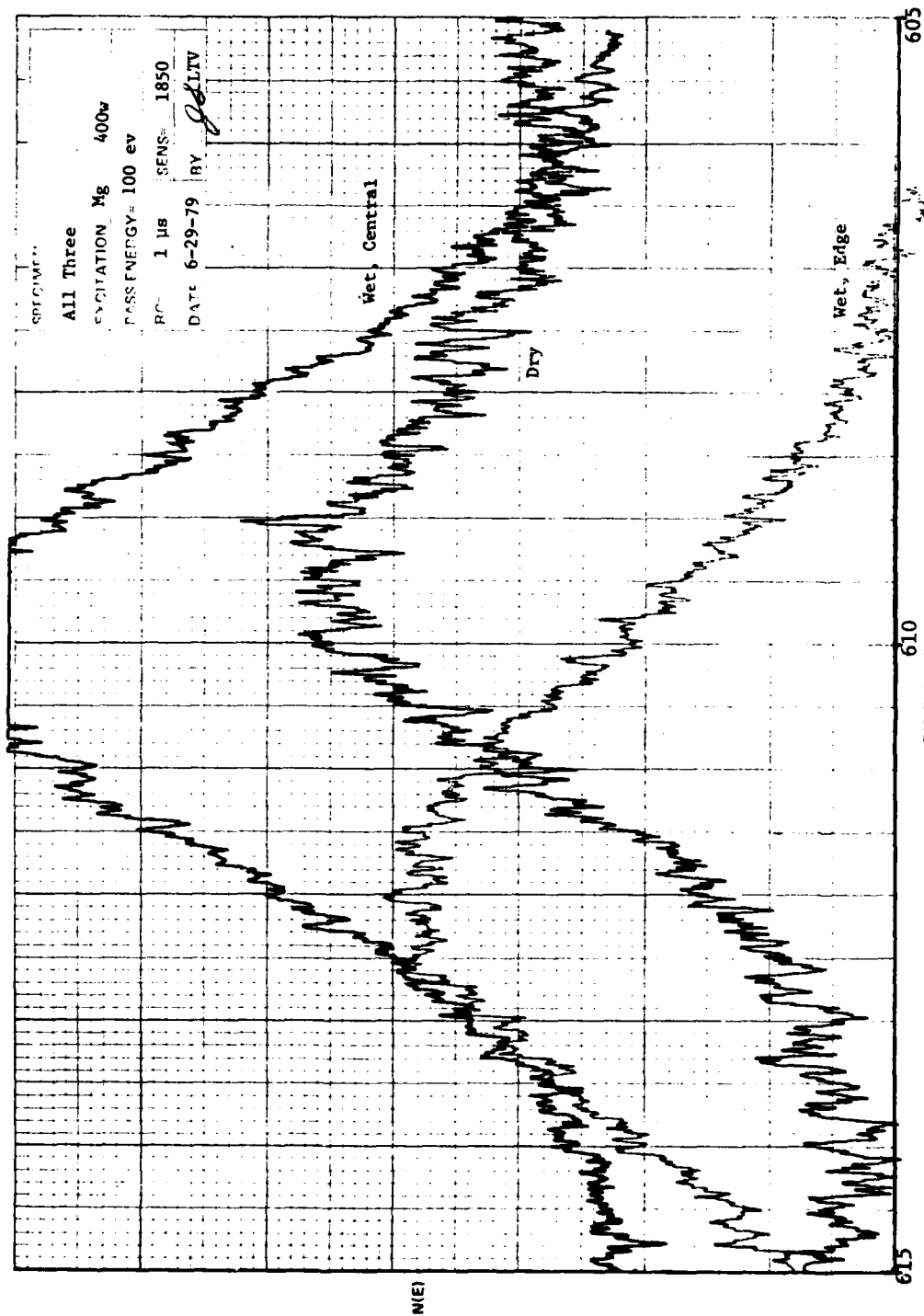


Figure A71 High Resolution XPS Spectra: C<sub>1s</sub> Peak for Different FM-73M Adhesive Conditions

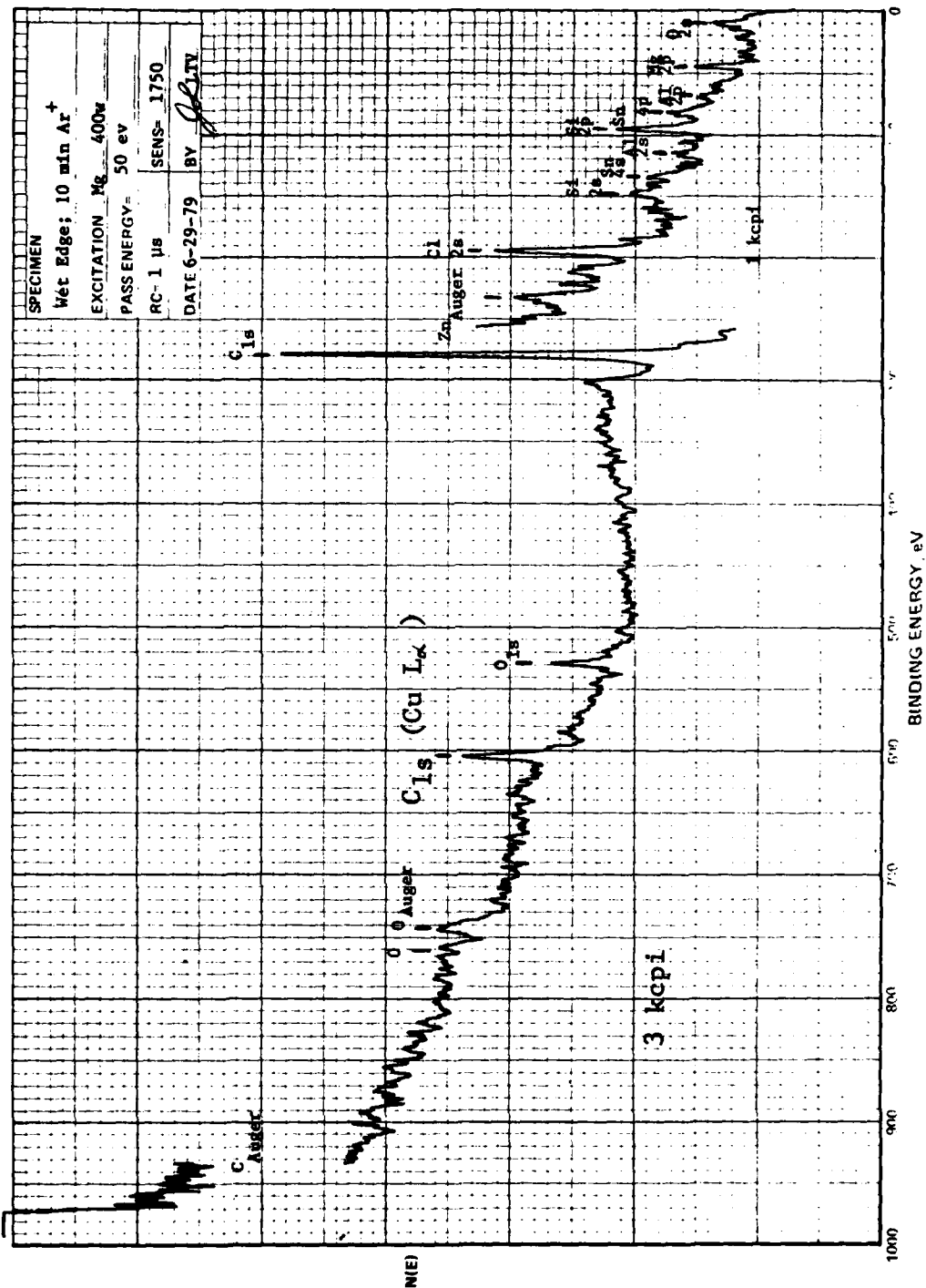


Figure A72 XPS Spectrum of 800 A-Sputtered Wet FM-73M Interlayer Edge

Appendix A

ON THE STRESS DISTRIBUTION IN THE  
ADHESIVE INTERLAYER OF A MODEL JOINT  
WITH RIGID ADHERENDS



# ON THE STRESS DISTRIBUTION IN THE ADHESIVE INTERLAYER OF A MODEL JOINT WITH RIGID ADHERENDS - W. G. KNAUSS

We assume that Poisson's ratio,  $\nu$ , is a constant and that only Young's modulus,  $E$ , is a (viscoelastic) time-dependent function,  $E(t)$ .

The formulation of the two-dimensional boundary value problem starts with listing the field equations. We consider all field functions to depend on the spatial coordinates  $x$ ,  $y$ , and time. Laplace transformation (indicated by a bar over the field quantity) introduces the Laplace parameter  $p$ .

$$\overline{\tau}_{ij,j} = 0 \quad \text{equilibrium equation} \quad (1)$$

$$\overline{\sigma}_x = \overline{\phi}_{,yy} \quad \phi = \text{Airy stress function} \quad (2)$$

$$\overline{\sigma}_y = \overline{\phi}_{,xx}$$

$$\overline{\tau}_{xy} = -\overline{\phi}_{,xy}$$

$$\nabla^4 \overline{\phi} = 0 \quad \text{compatibility equation(s)} \quad (3)$$

$$\bar{\epsilon}_x = \frac{1-\nu^2}{pE} \left( \bar{\sigma}_x - \frac{\nu}{1+\nu} \bar{\sigma}_y \right) \quad \text{stress-strain relations (plane stress)} \quad (4)$$

$$\bar{\epsilon}_y = \frac{1-\nu^2}{pE} \left( \bar{\sigma}_y - \frac{\nu}{1+\nu} \bar{\sigma}_x \right)$$

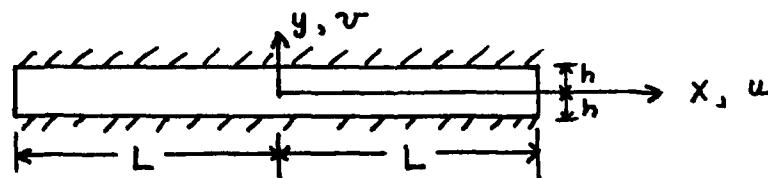
$$\bar{\epsilon}_{xy} = \frac{1+\nu}{pE} \bar{\tau}_{xy}$$

$$\bar{\epsilon}_x = \frac{\partial \bar{u}}{\partial x} \quad \text{strain-displacement relations} \quad (5)$$

$$\bar{\epsilon}_y = \frac{\partial \bar{v}}{\partial y}$$

$$2\bar{\epsilon}_{xy} = \frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{v}}{\partial x}$$

Besides satisfying these field equations the stresses and displacements must satisfy the boundary conditions.



We assume rigid adherends

$$\text{on } y = +h \quad u = u_0(t); v = 0 \quad \text{or} \quad \bar{u} = \bar{u}_0; \bar{v} = 0 \quad (6a)$$

$$y = -h \quad u = -u_0(t); v = 0 \quad \text{or} \quad \bar{u} = -\bar{u}_0; \bar{v} = 0$$

$$\text{on } x = L \quad \sigma_x = \tau_{xy} = 0 \quad \text{or} \quad \bar{\sigma}_x = \bar{\tau}_{xy} = 0 \quad (6b)$$

$$x = -L \quad \sigma_x = \tau_{xy} = 0 \quad \text{or} \quad \bar{\sigma}_x = \bar{\tau}_{xy} = 0$$

Because Equation (3) is independent of material properties, we find  $\bar{\phi}$  independent of  $\bar{E}$ . We find the stresses  $\bar{\sigma}_x$ ,  $\bar{\sigma}_y$ ,  $\bar{\tau}_{xy}$  from Equation (2) in terms of  $\bar{\phi}$ , and these are independent of  $\bar{E}$ . Define  $\nu' = \frac{\nu}{1+\nu}$ ,  $E'(t) = \frac{E(t)}{1-\nu^2}$ , or  $\bar{E}' = \frac{\bar{E}}{1-\nu^2}$ .

Then we find from Equations (4) and (5) that

$$\begin{aligned} \rho \bar{E}' \frac{\partial \bar{u}}{\partial x} &= \bar{\sigma}_x - \nu' \bar{\sigma}_y \\ \rho \bar{E}' \frac{\partial \bar{v}}{\partial y} &= \bar{\sigma}_y - \nu' \bar{\sigma}_x \end{aligned} \quad (7)$$

In order to be able to apply the boundary conditions of Equation (6a), we integrate Equation (7) to find

$$\rho \bar{E}' \bar{u} = \bar{F}(x, y, \rho, \nu) = \int (\bar{\sigma}_x - \nu' \bar{\sigma}_y) dx + \bar{f}(y, \rho) \quad (8a)$$

$$\rho \bar{E}' \bar{v} = \bar{G}(x, y, \rho, \nu) = \int (\bar{\sigma}_y - \nu' \bar{\sigma}_x) dy + \bar{g}(x, \rho) \quad (8b)$$

The  $\bar{F}$  and  $\bar{G}$  denote functions of the coordinates  $x$ ,  $y$ , of the constant Poisson's ratio,  $\nu$ , and the Laplace parameter  $\rho$ . The latter dependence enters through the dependence of the stresses on  $\rho$ , not directly through the material property

$E$  or  $\bar{E}$ . The functions  $\bar{f}(y, \rho)$  and  $\bar{g}(x, \rho)$  are determined by substituting Equations (8a) and (8b) into:

$$\frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{v}}{\partial x} = \frac{2(1+\nu)}{\rho \bar{E}} = \frac{1}{\rho \bar{E}'} \frac{2 \bar{T}_{xy}}{1-\nu} \quad (9)$$

In this latter substitution operation, the modulus  $\bar{E}'$  will drop out. So far then, the stresses are not functions of  $\bar{E}'$ .  $\bar{E}'$  enters through the boundary condition given by Equation (6a). From Equation (8) we have

$$\begin{aligned} \bar{u} &= \frac{1}{\rho \bar{E}'} \bar{F}(x, y, \rho, \nu) \\ \bar{v} &= \frac{1}{\rho \bar{E}'} \bar{G}(x, y, \rho, \nu) \end{aligned} \quad (10)$$

Boundary condition of Equation (6a) requires

$$v(x, \pm h, t) = 0$$

and therefore  $\bar{G}(x, \pm h, \rho, \nu) = 0$  and does not introduce  $E'$  or  $\bar{E}'$ . The conditions

$$\bar{u}_0 = \frac{1}{\rho \bar{E}'} \bar{F}(x, +h, \rho, \nu) \Rightarrow \rho \bar{E}' u_0 = \bar{F}(x, +h, \rho, \nu) \quad (11a)$$

$$-\bar{u}_0 = \frac{1}{\rho \bar{E}'} \bar{F}(x, -h, \rho, \nu) \Rightarrow \rho \bar{E}' u_0 = -\bar{F}(x, -h, \rho, \nu) \quad (11b)$$

say that  $\bar{F}$  is an odd function in  $y$ . Let  $\bar{F}_*$  be that odd function so that Equation (11b) is satisfied if Equation (11a) is satisfied. But  $\bar{F}_*$  depends on  $\bar{E}'$ , but only in a product form with  $\bar{u}_0$ . Thus the function  $\bar{F}_*$  is proportional to the product  $\bar{E}'\bar{u}_0$ . But this makes the whole solution proportional to the product  $\bar{E}'\bar{u}_0$ .  $u_0(t)$  or  $\bar{u}_0$  is, of course, the only forcing function and thus all stresses and displacements are proportional to it. If  $\bar{F}_*$  is proportional to  $\bar{E}'\bar{u}_0$  then from Equation (10) it follows that  $\bar{u}$  is proportional to  $\bar{u}_0$ , but not to  $\bar{E}'$ , nor to  $\frac{1}{\bar{E}'}$ . Furthermore, the stresses are also proportional to  $E'$  or  $\bar{E}'$ , and  $E'$  (or  $\bar{E}'$ ) is factored out of the functions for the stresses.

We now turn the problem around in the sense of prescribing the loading on the rigid adherends and ask for the resulting creep displacement.

Choose the displacement function  $u_0(t)$  such that

$$u_0(t) = u_0 D'(t) \quad \bar{u}_0 = u_* \bar{D}'(t)$$

$$\text{then } D'(t) = (1-\nu^2) D(t) \quad (\text{creep compliance in tension})$$

$$\text{and } \rho^2 \bar{D}' \bar{E}' = 1$$

$$\text{then } \bar{u}_0 \bar{E}' = u_* \Rightarrow \bar{u}_0 = \frac{u_*}{\bar{E}'}$$

Since the stresses were proportional to  $\bar{E}'\bar{u}_0$ , this choice of  $u_0(t)$  now makes the stresses independent of  $\bar{E}'$ . In fact, the stresses will be time-independent and the displacements will increase with time as the creep compliance.

Integrate  $\tau_{xy}$  over  $x$  along  $y = \pm h$ . That total load is also time independent. We have now essentially the bond problem. A constant load is accompanied by a time-independent stress distribution and by time dependent creep displacements.

This was possible only because we assumed Poisson's ratio,  $\nu$  = constant, and because we could prescribe, a priori, the displacements at  $y = \pm h$  on rigid boundaries.

## Appendix B

### PUBLICATIONS

The following presentations, technical reports, and publications cover work carried out under this contract. They were either prepared or appeared publicly during the reporting period:

ASM Metals Handbook, Chapters on "Inspection by Optical and Ultrasonic Holography", Vol. 11, Nondestructive Inspection and Control, Eighth Edition, 1976, pp. 198-233.

"Fatigue Behavior of Adhesively Bonded Joints", 1st PABST Industry Review, Long Beach CA, 5-7 October 1976.

"Environmental Effects in Adhesively Bonded Joints", General Dynamics' Fort Worth Division Report ERR-FW-1768, 16 December 1976.

"Fatigue Behavior of Adhesively Bonded Joints:", 2nd PABST Industry Review, Long Beach, CA, 14-16 September 1977.

"Fatigue Behavior of Adhesively Bonded Joints", 1st AFOSR-AFML "Workshop on Significance of Time-Dependent Behavior of Structural Adhesives", Grand Prairie, TX, 4-5 December 1977.

"Joint Failure Model - Environmental Effects", General Dynamics' Fort Worth Division Report ERR-FW-1850, 16 December 1977.

"AES/ESCA analyses of Surfaces of Adhesively Bonded Joints", 2nd Southwest Electron Spectroscopy Users' Conference, Texas A & M University, College Station, TX, 2 June 1978.

"Fatigue Behavior of Adhesively Bonded Joints", 3rd PABST Industry Review, Long Beach, CA, 27 September 1978.

"Environmental Serviceability of Bonded Joints", General Dynamics' Fort Worth Division Report ERR-FW-1963, 22 December 1978.

"Behavior of Adhesively Bonded Joints Under Cyclic Loading", Seminar to AFFDL, W-P, OH 10 January 1979.

"Fatigue Behavior of Adhesively Bonded Joints", Meeting of the Adhesion Society, Savannah, GA, 12-14 February 1979.

"Fatigue Behavior of Adhesively Bonded Joints", and Session Chairman, Workshop on Mechanics of Adhesively Bonded Joints", AFML/MBC, W-P AFB, OH, 6 March 1979.

Lecture 4, "Behavior of Adhesively Bonded Joints Under Cyclic Loading", AGARD-NATO Lecture Series No. 102 on "Adhesive Bonding and Preparation for Bonding", Oslo, Norway, 2-3 April 1979; The Hague, The Netherlands, 5-6 April 1979; and W-P AFB, OH, 16-17 October 1979; also AGARD publ. AGARD-LS-102 March 1979.

"On the Time-Dependence of Poisson's Ratio of a Commercial Adhesive", with W. G. Knauss, J. Adhesion, 1979.

"Fatigue Mechanisms in Adhesively Bonded Joints", AFOSR Seminar at Texas A & M University, Mechanics and Materials Research Center, March 27, 1980.

"Fatigue Behavior of Adhesively Bonded Joints", 25th National SAMPE Symposium & Exhibition, San Diego, CA, May 6-8, 1980.

"Fatigue Mechanisms in Adhesive Joints", Developments in Adhesives - 2, edited by A. J. Kinloch, Applied Science Publishers, 1980.



